BOSTON UNIVERSITY

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Dissertation

THE DEVELOPMENT AND USE OF INDIFFERENCE CURVES IN ECONOMIC DEMAND ANALYSIS

by

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OUTLINE

CHAPTER I

INTRODUCTION

In the history of economic thought the development of the theory of consumer demand is associated largely with neo-classical thought. It is not that the consumer aspect was neglected by the early classicists, but rather that it was relegated to the background, to such an extent that it became almost insignificant in their account of things. Cost and labor value theories were stressed and worked out in numerous directions. But these involved great difficulties, many of which could not be solved. A new approach seemed necessary, and fortunately it was afforded by the utility theory. The introduction of the utility approach was of some aid in the old problems, but it probably raised more new problems than it solved old. However this may be, there were certain advantages to it. Theoretically it served to broaden the economic perspective. Cost of production, whether in terms of labor value or money value, was no longer the determining element in value. And today in spite of the limitations to the theory of consumer behavior, it is at last recognized that value is not a one-sided phenomenon. As Marshall put it, it is a "scissors,"

with the blades of production and consumption both vital to the whole process.

The decisive factor in the neoclassical broadening of the economic perspective lies in the use of the utility concept. Utility, as first introduced, was essentially introspective. But in the course of its development it has been gradually purged of those inherent connotations, largely introspective, which were objectionable. The psychological, ethical, and hedonistic aspects of the concept were over a period of time done away with. The remaining concept has thus become a far more acceptable doctrine from a theoretical point of view, but from a practical point of view it is not a more useful one.

The concept originally stressed the utilitarian and hedonistic aspects. It depended chiefly on introspective magnitudes. But little by little there arose a skepticism about the measurement of utility introspectively. It became apparent that this precluded the existence of a standard unit. Also along with this it was recognized that the absolute measurement of utility was not an essential prerequisite to a theory of consumer's behavior. What was needed was merely a system of assumed preferences involving measurement of "more or less" but not "how much." This manifested itself in the development of the indifference analysis; indeed, the indifference analysis is considered by some as one of the turning points in the development of the theory of consumer

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behavior.

The chief purpose of this paper is to recount the development of indifference analysis and to evaluate it theoretically in the light of the present state of the theory of consumer behavior. It should be understood clearly beforehand that the indifference curve is merely a geometric device which serves as an analytical tool and that from the use of this tool no unusual practical results have resulted. For the person who is seeking purely practical results this paper is of little value. From a theoretical point of view, however, I feel something has been added. If indifference analysis did nothing more than limit the concept of utility, it would have served its purpose, but it has done more than that. It has set up new limitative hypotheses from which a system of reasoning has been built. It has limited the type of empirical data to be taken into consideration. And, as a result of this, there has been a broadening, however small, of insight into the relationships on the demand side of the economic equation.

Ordinarily there would be little need for a paper of this sort. Various aspects of the subject are covered in numerous modern texts and it would therefore seem that the development material was readily available. Unfortunately this is not the case. The early work appears mostly in academic journals. Some of it has been done in foreign tongues and has not been translated into the English. Generally speaking, the material

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is accessible only with great difficulty and in discontinuous form. Consequently it is not as well known as it should be and its influence has not been widespread. In the following pages, therefore, I shall attempt to set out a concise picture of the development of this aspect of demand theory. More than that, I shall attempt to evaluate it as a tool, for, as such, its popularity has reached the point where an inquiry of this type is necessary even if we should end by completely denying its usefulness.

The development of the indifference curve was the outgrowth of pioneer thought in the theory of consumer behavior. This thought has extended in two directions. Primarily it embraced the logical establishment of individual and group demand curves. But along with this, it has included a specific problem incidental to these curves, the problem of related goods. Unfortunately in the developmental phase these areas of thought have been treated together and as a result there has been much unnecessary confusion. Furthermore the treatment of the specific problem has attracted attention far out of proportion to what it deserves. The reason for this probably lies in the nature of the device itself and the suggestive possibilities which it infers. However, the two problems are definitely related; yet also in a narrow sense they can be separated. Our task is to present them in their related sense and also to separate them. To this end I shall treat the development somewhat

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differently from that which for the most part actually took place. The first half of this paper will treat the device in its broad aspects. It will include the establishment of the indifference curve and its use in the theoretical construction of individual and group demand curves. In the latter half the problem of related goods shall be presented. This will involve repeated reference to material covered in the first half, but for the sake of clarity the reader is asked to make this reference. It is hoped that two complete pictures and their points of similarity and separation will thereby be attained.

Throughout the paper there will be found from time to time the resort to mathematical terminology. This I have attempted to keep to a minimum for two reasons. First while I possess a sufficient knowledge of mathematics to follow with some labor the mathematical reasoning, I do not dare use my mathematics as a tool of thought. Secondly, while I recognize mathematics as a great aid in economic reasoning, I feel that there is something to be gained by translating the mathematics back into literary thought. This is often overlooked by mathematical economists and as a result much of their work has become a riddle to all except a very few. This is in a large measure unnecessary. Whatever can be said mathematically can be stated in ordinary language and it is quite probable that many of the ambiguities which enter through the use of

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mathematics may be eliminated by resorting back to simple terms. One should be a check on the other. However this may be, the subject matter has been worked up by its originators mathematically and therefore some reference to mathematic concepts cannot be avoided.

One further point before we proceed. The indifference curve has already been referred to as a geometric device. The geometric implications are important, but this is of interest to us only in so far as it bears economically on the theory of consumer behavior. This limitation is necessary, for the device has been used in other aspects of economic thought. Under the name of iso-product curves the same device has been used on the productive side of the economic equation. Indeed some of the early work in this field was done by the same men who worked on the problems of demand. As economists they recognized that there was an application and they attempted to work this out. For our purposes, however, the term indifference curve will refer to the device as used on the subjective side and we shall accordingly limit our discussion to that end.

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CHAPTER II

HISTORICAL DEVELOPMENT

Edgeworth

There is no single formula which can adequately describe the forces which operate in the development of economic technique. It is customary, however, to ascribe exclusive importance to the work of individual men. To the extent that much of this work displays keen brilliancy, it is not to be understressed, yet at the same time care should be taken lest it be overstressed. It is well known today that no great mechanical achievement is the result of a single invention, but rather the accomplishment of a group of inventors.[1] Ideas accumulate; inventions are put together, the final results depend on the previous cumulative groundwork. A very similar pattern of forces can be said to operate in the development of economic technique. Therefore while we give credit where it is due, while we recognize the brilliant mind, it is more important to stress the making and remaking of the original ideas, which have been the groundwork of the final achievement.

[1] P. A. Usher, Industrial History of England (New York: Houghton Mifflin Company, 1924), p. 251.

Indifference analysis is one of the tools in economic technique. In a sense it is a mechanical achievement, which has opened up new vistas in the field of economic analysis. In its infancy, it was an insignificant device used to clarify the difficulties of a particular problem, but it has been so worked and reworked that today the final tool is very far removed from the original.

F. Y. Edgeworth is usually referred to as the "inventor" of the indifference tool. We first find the notion in his Mathematical Psychics.[1] Here the problem is to determine the extent of indeterminateness in the case of a contract between two people where no competition prevails. Edgeworth treats the problem mathematically and it is quite possible that the idea of the indifference curve came from the mathematical treatment. For our immediate purpose, however, we shall dispense with the mathematics since explanation can be made otherwise. In setting up his problem Edgeworth falls back on the work of Jevons. The cardinal thought of Jevons' theory is that value in exchange corresponds to the utility of the least useful portion of the commodity. When exchange takes place, the price must be such as to equate the "final utilities" of the exchangers. To illustrate this Jevons worked up his now

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famous equation of exchange.[1] But Jevons' problem was not that of Edgeworth, for he assumed that competition prevails. His formula of exchange applies not to bare individuals, but to those clothed with the properties of a market.[2] Edgeworth starts from Jevons' formula, but he changes the hypothesis so that the problem considered is that of pure barter between two individuals. He then proceeds to show that in such a case a contract without competition is indeterminate. Where perfect competition prevails the contract is perfectly determinate. As competition becomes more or less perfect, the contract becomes more or less indeterminate.

To illustrate this he sets up his indifference curves. In Diagram 1, X denotes the amount of one good given up by A and received by B; Y denotes the amount of another good given by B and received by A. The curve OR is an indifference curve for Y. It is the locus of all points which give to Y the same satisfaction. Similarly OS is the indifference curve of X. Now the line of force indicating the direction X or Y will care to move, will be perpendicular to the indifference curve. (Edgeworth calls these lines of preference.)[3] The process of exchange will continue so long as the gain in satisfaction of

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[1]W. S. Jevons, Theory of Political Economy (London and New York: Macmillan and Company, 1871), p. 101.

[2]Edgeworth, op. cit., p. 31, footnote 1.

[3]Ibid., p. 22.

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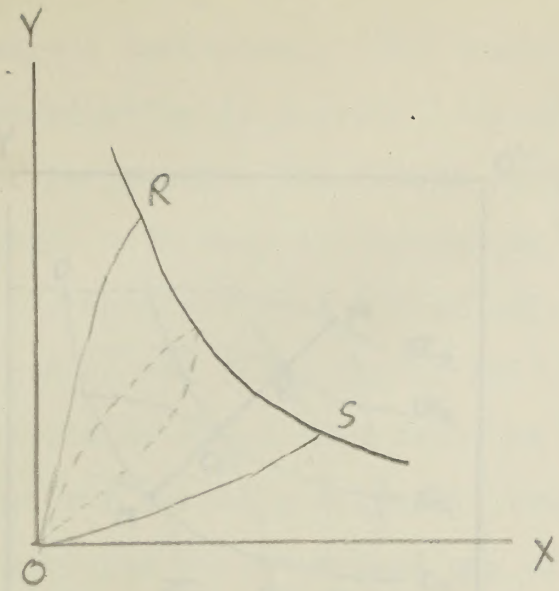

DIAGRAM 1

both A and B is positive. This condition holds true for any combination up to the line RS which is designated as the contract curve. Beyond the contract curve satisfaction will decrease. Barter therefore can take place only on the contract curve RS, but at what point the bartering will come to a stop cannot be predicted. The position of equilibrium may be described as indeterminate,[1] and the essential condition of this indeterminateness is the absence of competition.

This whole idea may appear clearer if we translate it into terms of the indifference systems as they are generally used. In Diagram 2 the indifference curves of A are represented by the curves I_a, II_a, III_a, etc., those of individual B represented by I_b, II_b, III_b. They are drawn with reference to axis O'X' and O'Y' which have been rotated 180 degrees. The location of each axis is determined by the fact that OA and OB are the total amounts of the two commodities which the

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[1]See discussion in F. Y. Edgeworth, Papers Relating to Political Economy (London: Macmillan and Company, 1925, published on behalf of the Royal Economic Society), Vol. II, pp. 315-319.

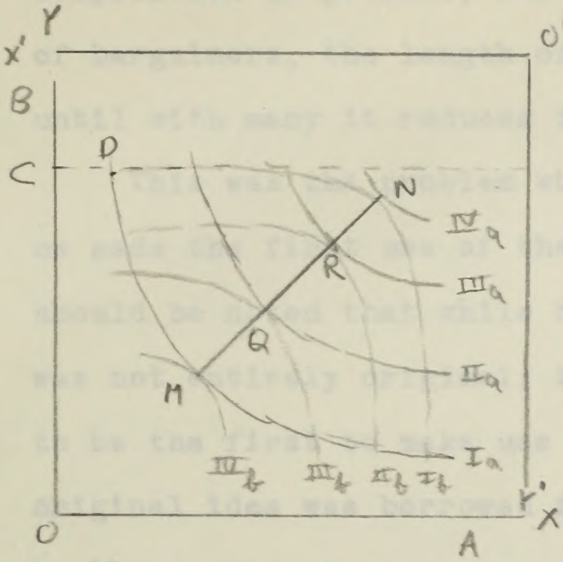

DIAGRAM 2

individuals together possess. Now if A possesses OC of Y and CD of X he will not exchange on a curve lower than I_a. Similarly B will not exchange on a curve lower than I_b. But if we assume that the curves indicate degrees of utility,[1] then exchange will take place where the marginal utilities are equal (Jevons above) or where the indifference curves meet at M, Q, R, N. MN represents all possible positions of equilibrium. Every point on the curve represents a final position equilibrium, where the marginal utilities being equal it is possible for a contract to take place. The position of the ultimate contract will be nearer to M if B is the more resourceful bargainer and nearer to N if A is the more resourceful. It is shown, therefore, that the essential condition of the indeterminateness does not depend on the equating of final utilities. This is

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[1]The early analysis was based on the utility concept and we resort to it here to elucidate this point. This discussion is based on that of George J. Stigler, in his <u>Theory of Price</u> (New York: Macmillan Company, 1946), pp. 79-81. Here Stigler discusses the question without reference to the utility concept. He merely assumes an ordinal preference field, which is the generally accepted method today.

merely incidental. The important factor is whether or not competition is present, for with the increase of the number of bargainers, the length of the contract curve decreases until with many it reduces to a single point.

This was the problem which Edgeworth attacked and in which he made the first use of the indifference curve. However, it should be noted that while he did an excellent job, his work was not entirely original; that is in so far as it is supposed to be the first to make use of the indifference device. The original idea was borrowed from Marshall and applied by Edgeworth.

Marshall developed the device to simplify the intricacies of foreign trade. He was interested in showing the gains resulting from foreign trade and also the tendency for it to gravitate towards a stable equilibrium, but he had to do away with changing monetary standards. He therefore translated this trade into terms of goods only and constructed a curve showing the amount of one good a country is willing to exchange for a given amount of another.[1] For example in Diagram 3, OE is drawn such that country A is willing to give PN of cloth in return for PM of linen. The curve actually equates the "national" utilities of PN cloth with PM linen as long as point P

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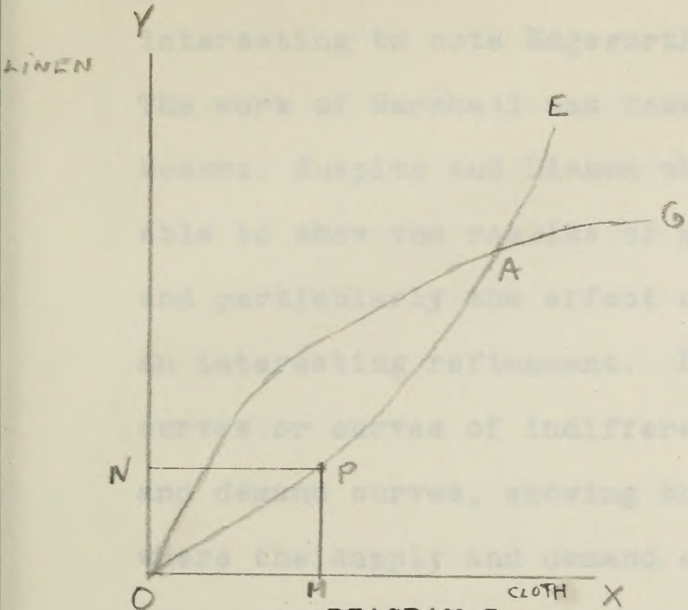

DIAGRAM 3

lies on curve OE and is on, in our sense, an indifference curve. Marshall of course did not think of it in just this manner. He was interested in showing the tendency for stable equilibrium at point A[1] and also the different shapes which the curves might possibly take under given conditions. For his purpose the device was perfect.[2]

Edgeworth in his work borrowed this device, but in so doing he changed it about almost entirely. He applied it to the problem of exchange, particularly to the subjective utility analysis of value, and more important, he developed the proper mathematical basis and gave the curve the name by which we now know it. Whereas Marshall made no further use of the device, Edgeworth also used it in the fields of foreign trade and taxation. Therefore, while it may not be right to attribute complete originality to Edgeworth, he certainly should be recognized for his originality of application and development.

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[1]<u>Ibid</u>., p. 20.

[2]See Edmund Whittaker, <u>A History of Economic Ideas</u> (New York: Longmans, Green and Company, 1940), pp. 448-449, for a discussion of Marshall's use of this device.

In connection with Marshall's construction above, it is interesting to note Edgeworth's application in the same field. The work of Marshall was taken up and developed in Germany by Messrs. Auspitz and Lieben who by using Marshall's device were able to show the results of gains and losses in foreign trade and particularly the effect of import taxes.[1] Edgeworth added an interesting refinement. He set up his international utility curves or curves of indifference, and then allowed for supply and demand curves, showing that equilibrium would be obtained where the supply and demand curves, at the point of intersection, meet the contract curve. This would correspond to the dotted lines in Diagram 1. What is important in this discussion is that Edgeworth calls attention to the fact that this is a device similar to his preference curves previously developed. Also not to be overlooked is the fact that Edgeworth refers to these curves as "utility curves."[2]

Fisher

At about the same time Edgeworth was doing his work in England, an American, Irving Fisher, was working along the same lines. Fisher's work is contained in his Mathematical Investigations in The Theory of Value and Prices which was

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[2] Edgeworth, _Papers Relating to Political Economy_, _op. cit._, p. 295.

presented as a part of his doctoral requirements. Unfortunately the first part of the work utilizes by way of illustration several intricate mechanical devices so that it becomes unnecessarily complicated and loses much of its value. The second part of the work contains the discussion on related goods to which Fisher turned after he developed the indifference system; indeed, the discovery of this device allowed Fisher to take up that problem.

There has been some question as to whether the device was first developed by Edgeworth or Fisher. From Fisher's introduction it is evident that both came upon it simultaneously, each independent of the work of the other. Fisher says,[1]

> Three days after part II was finished I received and saw for the first time Prof. Edgeworth's "Mathematical Psychics." I was much interested to find a resemblance between his surface on page 21 and the total utility surfaces described by me.

It is evident, then, that each worked independently, yet as we proceed it will become apparent that in Fisher's work the indifference device was very much more developed so that he was able to use it for problems which Edgeworth's development did not allow.

Both Edgeworth and Fisher were to a large extent dependent on the work of Auspitz and Lieben in Germany. Fisher in his

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[1]Irving Fisher, Mathematical Investigations in The Theory of Value and Prices (New Haven: Connecticut Academy of Arts and Sciences, 1892-1895), Vol. 9, p. 4.

preface acknowledges their influence.[1] The work of these latter two covered a period of ten years during which time several chapters were published previous to the main work. The ingenuity of these men is indeed admirable. It is incredible how close they came to developing the indifference device. Yet one is dismayed that after coming so close they did not take the final jump.

The problem they attacked was the old problem of value. They sought to examine the factors determining price and to show the relationships between price itself and the factors determining it. One of these factors, of course, was utility, and to show the relationship between utility and price, they set up a utility curve which theoretically resembles an indifference curve. This curve arises convex from the origin and finally drops below the X axis. In describing the utility which this curve portrays they say,

> The utility is measured by the greatest effort or the greatest sum of money which the consumer can expend for this commodity without disadvantage; the consumer will seek no advantage in this expenditure so that he is entirely indifferent (gleichgiltig) whether or not he buys that quantity of consumable good.[2]

Here is perhaps the first suggestion of the idea of indifference as applied to utility, but it is still not indifference in the current use of the concept. The curve is

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[1] _Ibid_., p. 3.

[2] Auspitz and Lieben, _op_. _cit_., pp. 8-9.

essentially a utility curve very similar to Marshall's curve. At each point on the curve a certain utility is derived from a quantitative expenditure, and if the individual attains that utility from the given expenditure, then he is indifferent as to whether he buys the quantity of the given good, or a quantity of some other good which yields a similar utility. The germ of the indifference concept is here, but it is still in very rough form.

In their method Auspitz and Lieben are very helpful; indeed it appears that they pave the way. Their real contribution lies in the use of the three dimensional coordinate system. After examining the different variable factors which go to make up price, and the many possibilities of variation, they attempt to bring these together, through the use of the spatial coordinate system. They establish what is called a satisfaction surface (Befriedigungsfläche).[1] This surface appears in Diagram 4.[2]

The X axis represents quantity of the given good. Y represents consumer expenditure, and Z utility. The surface then indicates the total satisfaction of a given good in relation to all possible prices and quantities of that good. But it is assumed that the price of all other goods, B....N, are

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[1]Ibid., Anhang II, pp. 484-512.

[2]Ibid., p. 500. To simplify I have shown only one-half of their surface.

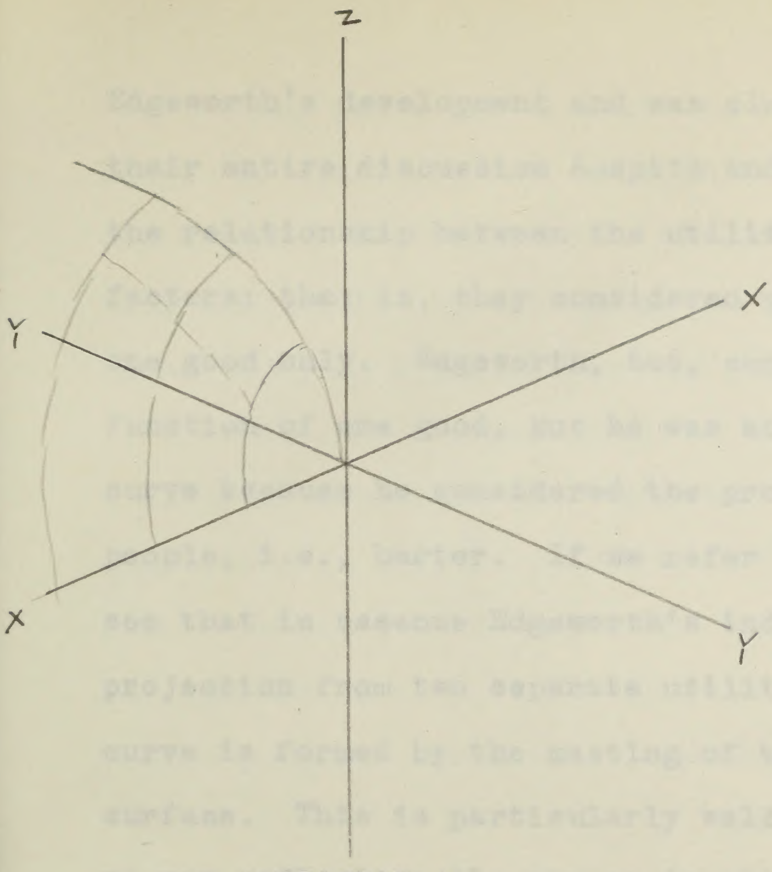

DIAGRAM 4

given and do not vary. What they actually establish, then, is a total utility surface for a single good. This is very similar to the total utility surface which is used by Fisher which shall be examined below. However, as will be seen, Fisher's derivation of the surface is slightly different.

The method which they now pursue is that of projecting onto this surface planes parallel to each axis so that the contours of the surface become visible. Fisher uses a similar procedure, but he goes even further by projecting the contours onto the zero plane ,thereby doing away entirely with the third dimension. Auspitz and Lieben did not make this final step, but there is no question that as far as they did go, they exerted a strong influence on Fisher's treatment.

The development of the satisfaction surface was a great forward stride, but there was still lacking in the reasoning of Auspitz and Lieben that extra twist which would have produced the indifference curve. This was somewhat made up in

Edgeworth's development and was clearly seen by Fisher. In their entire discussion Auspitz and Lieben considered only the relationship between the utility of one good and other factors; that is, they considered utility as the function of one good only. Edgeworth, too, considered utility as the function of one good, but he was able to develop the contract curve because he considered the problem of trade between two people, i.e., barter. If we refer to Diagram 1 we can now see that in essence Edgeworth's indifference curves are the projection from two separate utility surfaces; the contract curve is formed by the meeting of the highest points on each surface. This is particularly well brought out by Stigler if we now reflect on the manner in which he inverted the axes to clarify this.[1] The work of Edgeworth, then, may be considered as in between that of Auspitz and Lieben, and Fisher. To the extent that he considered the problem of barter, he was able to develop the indifference device, but the fact that he considered utility as the function of one good only limited the use of the device.[2]

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[1]Stigler took this technique from Pareto. See V. Pareto, <u>Manuel</u> <u>d'Economie</u> <u>Politique</u> (Paris: 1909), p. 191.

[2]It will be noticed that Edgeworth's indifference curve proceeds from the origin out. The reason for this is precisely the fact that he considered utility as the function of one good only. Fisher considered it as the function of two goods and therefore his curves take the customary shape.

Fisher makes the application which the others failed to make. He considers two commodities A and B consumed by one individual. He then directs his individual to alter consumption combination by arranging the quantities of the two selected commodities in all possible ways without changing the quantities of all other commodities C, D, E, etc. The marginal utility of each then will vary not only in relation to its own quantity, but also in relation to the quantity of the other. Now if we allow OX to represent the quantities of B and OY the quantities of A, than any point P on Diagram 5 will represent a possible combination consumed by the individual. By

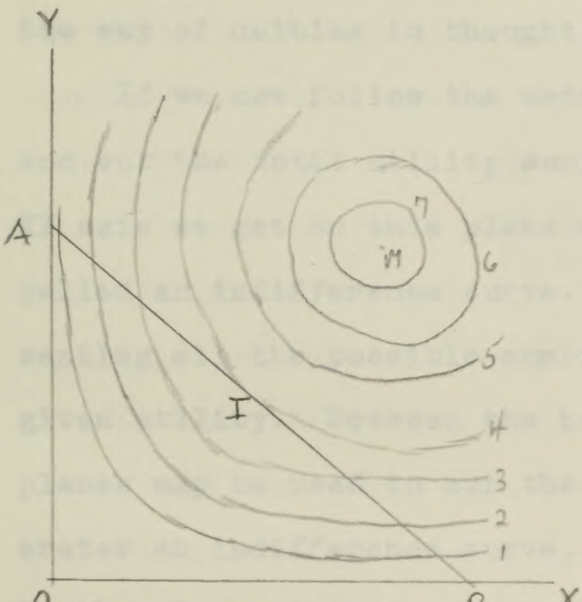

DIAGRAM 5

varying P, all possible combinations can be indicated. Now if we consider our axis in three dimensions by erecting a perpendicular at P, the length of which represents the marginal utility for A, that is the degree of utility of a small addition of A, B remaining the same, and if we allow P to take all possible positions, then the extremity of this perpendicular will generate a surface indicating the marginal utility for A. Similarly by holding A the same, and erecting another perpendicular, we will get another surface indicating

the marginal utility of B. Now mathematically marginal utility can be derived from total utility so that if we consider the two surfaces as derived from a third surface, we get at P a perpendicular which indicates the total utility of both A and B in combination and which forms a total utility surface. This surface because of the three dimensions will be convex, like a hill, rising to a maximum point. On this surface the slope of the plane parallel to A and tangent to the surface will give the marginal utility of A. Similarly the slope of the plane parallel to B and tangent to the surface gives the marginal utility of B. Thus the total utility surface provides the way of uniting in thought the two marginal utilities.[1]

If we now follow the method indicated by Auspitz and Lieben and cut the total utility surface by a plane parallel to the XY axis we get on this plane a curve which may be properly called an indifference curve. It is the locus of points representing all the possible combinations of A and B which have a given utility. Between the base and the tip, innumerable planes may be used to cut the surface. Each one, then, generates an indifference curve. If these are all projected on to the plane at the zero level we get a series of concentric

[1]We can see the relation between Fisher's total utility surface and that of Auspitz and Lieben if at point P we erect a perpendicular which moves parallel to A. B then will be held constant and the surface thus described will be the total utility surface for A, the surface described by Auspitz and Lieben. See Fisher, Mathematical Investigations in The Theory of Value and Prices, op. cit., p. 69.

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curves vanishing at M, the point of maximum utility. It has been assumed here that the surface has only one maximum, i.e., one peak. There of course may be several and these may lie anywhere within the XY axis. The important feature, however, is that at the zero level the absolute height of the primitive surface becomes of no consequence.

Referring to Diagram 5, let us now assume that the individual has an income of $25 per year to be spent on the two goods. If the price of A is 50 cents and B 25 cents, then the two simplest methods of spending the $25 are to spend it all on A or all on B; that is, to buy either 50 units of A or 100 units of B. If we mark off these quantities on the corresponding axis and connect the two points, then the line in between will indicate all possible consumption combinations of A and B which can be purchased at the given prices for the $25. Fisher calls this line AB, "the partial income line." The individual now must move along this line in a manner such that he can select that combination which is possible at his income and also which will yield the maximum total utility. This will occur at point I where the partial income line is tangent to an indifference curve. At any other point on the line the individual can attain greater utility by selecting a combination closer to I, but at I the individual gets the greatest utility for the given expenditure. We may say then that at I, while prices and quantities of other articles

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Referring to Diagram 3, let us now assume that the individual has an income of $25 per year to be spent on the two goods. If the price of A is 50 cents and B 25 cents, then the two simplest methods of spending the $25 are to spend it all on A or all on B; that is, to buy either 50 units of A or 100 units of B. If we mark off these quantities on the corresponding axis and connect the two points, then the line in between will indicate all possible consumption combinations of A and B which can be purchased at the given prices for the $25. Fisher calls this line AB, "the partial income line." The individual now must move along this line in a manner such that he can select that combination which is possible at his income and also which will yield the maximum total utility. This will occur at point I where the partial income line is tangent to an indifference curve. At any other point on the line the individual can attain greater utility by selecting a combination closer to I, but at I the individual gets the greatest utility for the given expenditure. We may say then that at I, while prices and quantities of other articles

remain the same, the individual is in equilibrium with respect to A and B.

The development of the indifference system by Fisher as described above produced a very important tool. Until this point utility had to be considered as the standard of measurement; that is, the utility of a commodity was the function of that quantity alone. But at this point he was able to consider the utility of one good as dependent on the quantity of another (the problem of related goods which we shall discuss later), and finally he was able to do away with utility almost entirely.[1] As we saw above the individual always gravitates towards equilibrium at point I. In so far as he seeks this equilibrium position he tries to seek the highest indifference curve. Fisher thought of the movement from one curve to another as the resultant of two forces, this resultant always being perpendicular to the indifference curve.[2] Thus, if lines of force are set up perpendicular to the indifference curves, then it is the direction of lines which determine equilibrium. "It makes absolutely no difference....what the length of the arrow is at one point compared with another";[3] that is, it

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[1] I say almost entirely because of the fact that in his discussion of related goods Fisher still held to the utility concept. He defines competing goods in terms of marginal utilities.

[2] Fisher, op. cit., p. 74.

[3] Ibid., p. 88.

makes no difference what is the height of the primitive surface as long as we know that at the zero level one curve is greater than another and that the individual must travel in the direction of the increase. Fisher concludes then, "Thus we may dispense with the total utility density and conceive the economic world to be filled merely with lines of force or 'maximum directions.'"[1]

It should now be apparent what the development of the indifference device has done for Fisher. Through it he has been able to rework the concept of utility and to discard from it the hedonistic subjective elements. His purpose has been to objectivize it as far as possible and to a very great extent this has been done. In determining the objective facts of price, certain attributes of utility can now be disposed of. For example, it is no longer essential that one man's utility be compared with another nor is it essential for the marginal utility of various consumption combinations to be compared. And most important of all, the whole concept of total utility can be done away with.[2] Under Fisher's system we are interested only in the <u>maximum directions</u> indicated by the quantitative

[1] Ibid.

[2] Fisher expresses this by saying in mathematical terms that even if marginal utilities for the same individual can be compared at one consumption combination to another, the total utility might not be integrable and if it were there would be no need to determine the constant of integration.

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factors and we can dispense with those hedonistic elements which are so objectionable.

Pareto

In our investigation so far we have traced the indifference device from its inception, through Edgeworth and Fisher. But this is only one of two directions in which the development took place. The other direction is to be traced through Edgeworth, Pareto and the Lausanne school, to Hicks. This latter course is by far the more well known, not so much for its originality in development, but rather for its specialization in the problem of related goods. However, the work of Pareto, the chief member of the Lausanne school, did produce some important developmental aspects and it is these to which we now turn.

We have seen how Fisher was determined to objectivize the theory of value. This was in effect one of the prime purposes of Pareto. In his earlier work, Cours d'Economie Politique, he constructs a theory of value entirely subjective, based on the notion of pleasure.[1] In the Manuel d'Economie Politique, a later work, he proceeds in the opposite vein. Subjective notions of pleasure he now says are not needed to erect a theory of economic equilibrium.[2] It is possible to

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[1] V. Pareto, Cours d'Economie Politique (F. Rouge, Editor; Lausanne: 1896), Vol. I, pp. 3-6.

[2] V. Pareto, Manuel d'Economie Politique (translated from the Italian by Alfred Bonnet, V. Giard, and E. Briere; Paris: 1909), p. 160.

start from a point of departure, quantitative and objective. The very fact that we can objectively determine quantities of goods which are consumed together is sufficient to establish a system of economic equilibrium. These objective quantities are what Pareto calls les combinations indifférentes. When an individual hesitates between two different combinations offered to him, this shows objectively that the two combinations are of the same importance to him. Thus what Pareto has done is to build his system on the assumption of lines of indifference. Starting thus with objective hypotheses, as he did, it should be possible to build a purely objective theory.

Pareto's analysis is somewhat similar to that of Fisher except that it is in terms of tastes and obstacles. His indifference curves are derived from a total utility surface, which he calls "la colline," the hill. In climbing the hill the individual follows "le sentier," the path, or in Fisher's terms, the partial income line, until he reaches the highest point possible, that is, the point of greatest total utility. At this point where the path is tangent to the indifference curve the individual will be at equilibrium, but this is an equilibrium with respect to tastes only. On the other side are to be considered the obstacles which the individual will meet. These are represented by a "colline des profits," similar to the utility surface. On this surface the producer will pursue that path which will maximize his profit, but the

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surface will also show that area in which the producer operates at a loss.[1] From this surface it is likewise possible to derive indifference curves (or as they are otherwise called, isoproduct lines) for the producer, and to show the point of equilibrium for the producer--with respect to obstacles.

Now to compare the equilibria, that is, to show equilibrium with respect to tastes and obstacles, Pareto develops a new curve. Referring to the indifference map (Diagram 6), we note that there will be several paths which are tangent to indifference curves. If we join the points of tangency we get curve AB which joins each point of equilibrium with respect to tastes. Pareto calls this the "line of exchanges."[2] There will be a similar line for producers which he calls the "line of greatest profit."[3] If these lines are superimposed on the same coordinate system, then when many producers contract with

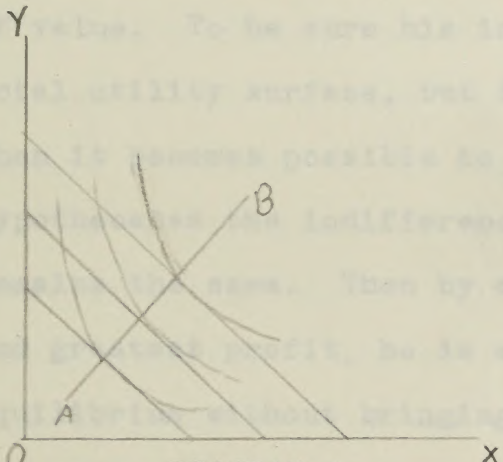

DIAGRAM 6

[1]A justified criticism at this point is that by explaining the indifference curves of the producer in terms of profit and loss, Pareto is using the notion of value to explain value. On this point see G. Pirou, Theories de L'Equilibrium (Paris: F. Lovitan and Cie, 1938), pp. 424-425.

[2]Pareto, Manuel d'Economie Politique, op. cit., p. 184.

[3]Ibid., p. 187.

many consumers, the point of equilibrium will be given by the intersection of the line of exchanges and the line of greatest profit.[1] Translating this we can say that this will be a point of stable equilibrium, for movement away from this point in any direction will immediately set up forces moving back to the original point.[2]

We can now see the originality in the work of Pareto. This lies essentially in his attempt to objectivize the theory of value. To be sure his indifference curves are taken from a total utility surface, but if they are taken as a hypothesis, then it becomes possible to reason objectively.[3] Pareto hypothecates the indifference curves. He assumes that price remains the same. Then by establishing the lines of exchange and greatest profit, he is able to indicate the point of stable equilibrium without bringing in any subjective factors. As he himself says in a footnote, "I consider the indifference curves as given and I deduce from them all that is necessary for the theory of equilibrium without having recourse to ophélimité."[4]

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[1]Ibid., p. 190.

[2]Ibid., p. 153. There will be of course different points of equilibrium, depending on the given conditions. For example, many individuals may be dealing with one producer or vice versa, or two isolated individuals may deal with each other. Pareto treats these different points of equilibrium. However, while they are of interest, they add nothing to the total development of the indifference tool.

[3]It is interesting to note Pareto's use of the indifference device in the subjective analysis. See Cours d'Economie Politique, op. cit., footnote, p. 35.

[4]Pareto, Manuel d'Economie Politique, op. cit., p. 169.

To the extent that Pareto did this, his theory was objective. However, it is to be noted that while his intentions were good, he did not carry out completely what he hoped to do. In so far as he developed the point of stable equilibrium, his argument remains objective. However, in analyzing tastes, he ran into the problem of related goods and in applying the indifference curves to this problem he was forced to fall back on the ophélimité concept, so that actually his argument is not as thoroughly objective as he presents it. However, the use of indifference curves as an assumption was a step forward and it paved the way for further development.

To Pareto this work was very important because it allowed an integration of the work of many of the theorists preceding him. The chief problem until now had been the determination of price or value in exchange, but his work changed this, because price and value now became part of the whole phenomenon of equilibrium. It was no longer a question of cause and effect relationship, one set of conditions being the sole cause for a given effect, but rather a question of balancing given conditions which are themselves interrelated. This was the result of what he called scientific economics (which made use of mathematics) as opposed to the previous purely literary economics. It made a change in the basic problem of economics because it now stressed equilibrium economics as a special aspect of value economics. Indeed that entity which the

literary economists called value does not exist by itself. As he says,

> If that vague and indeterminate thing which the literary economists call value has any relation with prices, it can be said that it depends on all circumstances, none excepted, which influence the determination of economic equilibrium.[1]

This change in the problem also helps us to better understand the work of previous economists and to put each in its proper place. Thus those who have regarded utility as the sole determining factor of value have considered only one side, that of tastes. In this class he places his predecessor at Lausanne, M. Walras, who, although he used the mathematical method and thereby laid the foundation of the work of Pareto, was not able to resist the pressure of general opinion which considered the causal determination of value, and thereby established utility as the sole determining cause of value.[2] Also in this class there would be considered Gossen, Jevons, and the Austrian school who stressed marginal utility. On the other side are to be considered those who stressed the obstacles. For example, Adam Smith and his cost of production theory; particularly Ricardo and Marx who established labor as the determining factor in value. All these theories today are not very useful to scientific economics because their approach is inadequate.

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[1] Ibid., p. 245.

[2] M. Walras, Elements d'Economie Pure (Lausanne: 1900).

Today only those which consider economic equilibrium and how it is born in the "opposition of tastes and obstacles" have any permanent theoretical value.

We have seen above how Fisher also tried to objectivize the theory of value and how he finally resolved economic motivation into lines of force. In many respects his results are similar to those of Pareto, but when we remember that his assumptions were framed on the basis of the utility idea, we see how much more developed is the work of Pareto. To be sure the technique of Fisher exerted a great influence on Pareto. Pareto acknowledges this indirectly.[1] But what has made the work of Pareto so much more important is that he has reoriented this technique within the framework of new definitions and has then proceeded to expand it to new heights. This is particularly true with respect to the development of the line of exchanges. The latter is a natural outgrowth of the work of Fisher, which as we shall see will be further expanded into a highly useful tool. From the point of view of mathematical technique it is one of the important achievements of Pareto and within the framework of his new definition it enabled him to work out his system of equilibrium.

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[1]Pareto, Manuel d'Economie Politique, op. cit., pp. 157, 159.

Marshall's Consumer Demand

Just previous to the above development the theory of consumer demand had been worked out on a different basis by Alfred Marshall, an Englishman. It will be instructive now to digress for the moment to compare Marshall's treatment with what we have so far traced, for his work is recognized as classic.

Marshall treated the theory of demand from a purely subjective point of view. Utility is measurable. It is a function of the quantity of the good possessed.[1] If a consumer with a given money income confronts a market for consumer goods, the prices of which are predetermined, then the question to be answered is how will he divide his expenditures among the different goods. Marshall assumes that goods will be available in sufficiently small quantities and that the individual acts as the rational man; that is, he will spend his money so as to receive the maximum amount of possible utility. Now, if utility is a function of quantity, then the more of a given good an individual possesses, the less will be the utility per additional unit (the law of diminishing marginal utility) and the actual amount purchased will be determined by the utility of the last unit. Translating in

[1]A. Marshall, Principles of Economics (London: The Macmillan Company, 1922), Ch. 3, Book II, p. 93.

terms of demand price, we may say, the larger the amount of a good that a person has, the less will be the price (purchasing power of money and also income being given) which he will pay for a little more of that good. Since each unit of expenditure will be spent so as to buy the greatest increment of utility, it must follow that when an individual faces a market of several commodities, the marginal units of each commodity purchased must yield the same marginal utility, or, expressing this in another manner, the marginal utilities of the various commodities purchased must be proportional to their prices.

Marshall's theory thus proceeds from the notion of maximizing total utility, the law of diminishing marginal utility, to the conclusion that the marginal utilities of various commodities will be proportional to their prices. This conclusion was by no means new. It had been pointed out by Jevons and also by the Austrian school. Both Fisher and Pareto were able to bring it out. But Marshall's explanation was simple and pointed; indeed his entire theory of demand was so much so that it became classic and went unquestioned for many years, even after the work of Pareto.

The work of Pareto in its preliminary phases is somewhat similar to that of Marshall. Pareto recognized the validity of the law of diminishing marginal utility on the basis of the utility concept, and he had somehow to achieve the results of this law without its actual use. This was accomplished by

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the use of the indifference map and the "sentiers." If we reflect on his equilibrium of tastes we find that the individual is at equilibrium only where the path is tangent to an indifference curve. It is at this point only that utility will be maximized for if he goes in either direction beyond this point, the consumer will arrive at a lower indifference curve. If we consider the path as the price line, which it essentially is, since its position and slope is determined by the given prices, then the point of tangency is the expression, in terms of indifference curves, of the proportionality between marginal utilities and prices. What Pareto actually did was to translate Marshall's theory of marginal utility into terms of indifference curves.

But in the process of translation Pareto accomplished something unusual, for he was able to arrive at the same result without the original subjective data. Marshall's theory implies the use of a utility surface in order to determine the quantities of goods an individual will buy at the various prices. Pareto's assumes only that the indifference map is known. This of course does away with a great deal of information conveyed by the utility surface, but it is the type of information which is objectionable. Whereas Marshall would say I choose A in preference to B because it has greater utility and I am therefore willing to pay more for it, Pareto would say, after observing that I choose A in preference to

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B, this shows that A is the more useful of the two. One is based on utility. The other is based on the observation of choice and deduces utility from this observation.[1] Pareto's approach thus does away with any subjective information. It tells us that one combination is preferred to another, but it does not say by how much. It does not involve measurement.

The difference in assumption between Pareto and Marshall is in a sense the difference between Pareto and Fisher. One is subjective, the other objective. But there is a further difference. Marshall considered only the utility of one good. Pareto and Fisher considered the utility of two goods. It is for this reason that Pareto's work went further than Marshall, for having considered one good in relation to another it became necessary to examine the different relationships; that is, it became necessary to treat the problem of related goods. Both Pareto and Fisher go into this. Marshall's work is singularly lacking in this respect.

The Indeterminateness of the Utility Function

We have seen how Pareto reasoned on the basis of the assumed indifference map. One of the chief factors which shunted him in this direction was the point, also made by Fisher, that the directions of indifference may not be

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[1] M. Murray in <u>Leçons d'Economie Politique</u> (translated from Italian by Pierre Boven; Paris: Payot and Company, 1920), supports Pareto in this respect, p. 113.

integrable.[1] Pareto took up this idea and from it he proceeded to show that even if it were possible to deduce a utility function from the directions of indifference, the utility function itself would be largely indeterminate. This proposition is stated in the text of the Manuel d'Economie Politique and supplemented in the appendix by a mathematical proof.[2] We shall set it out in words.

The propositions that the directions of indifference are not integrable, or that the utility function is indeterminate, are derived from the application of the calculus to economic reasoning. The meaning is not too difficult. If we assume a utility surface so that we know how much utility is derived from given quantities of goods in combination, then it is possible to derive from this a scale of preferences; that is, of any two combinations we can say whether the individual will prefer one to the other or whether he will be indifferent to either. This scale of preferences may be indicated by the indifference map. This was precisely the method used by Fisher when he derived his indifference curves from his total utility surface. Pareto also shows this when he proceeds from the hill to the indifference level. We see then that it is possible to go from the utility function (the total

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[2]Pareto, Manuel d'Economie Politique, op. cit., p. 159; Appendix, pp. 540-547.

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[1]See footnote 2, page 24.

[2]Pareto, *Manuel d'Economie Politique*. *op. cit.*, p. 159; Appendix, pp. 540-557.

utility surface is the utility function expressed mathematically) to the scale of preferences, but is it possible to proceed in the opposite direction? Given the scale of preferences, is it possible to derive the total utility surface?

The answer is negative, because a given utility surface is not the only surface from which the same scale of preferences may be derived. It is possible that a utility surface be twice the height of a given surface. Projecting the points of like utility on the second surface onto the base, will give the same scale of preferences, but the utility will be twice as great. In proceeding from the total utility surface to the preference scale, the index of utility is lost. One cannot then proceed in the opposite direction, because innumerable indices could be used. Mathematically we may say that the function with which we started is not the only one which will determine a given preference scale.[1]

Fortunately, though very much energy has been spent on this point from the point of view of mathematical curiosity, it is not of very great importance.[2] Whether or not one can

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[1]Mathematically the utility function may be differentiated so that it is possible to obtain the function of the indifference curve, but the function of the indifference curve is not integrable. It is not only that there is no constant of integration, but also that the utility function itself is indeterminate. See N. Georgescu-Roegen, "Pure Theory of Consumer's Behavior," Quarterly Journal of Economics, Vol. L, No. 4, August, 1936; Section V, also by the same author, "A Proposition of Pareto," Quarterly Journal of Economics, Vol. XLIX, No. 4, August, 1935.

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go from the preference function to the utility surface matters not as long as the essential condition that the individual will prefer a higher utility level to a lower one is preserved. For example, successive positions might be numbered 1, 2, 3, 4, 5, or 1, 3, 5, 7, or A, B, C, D, etc.--any increasing series that one may choose.[1] As long as the series is increasing and the individual seeks the highest possible level, then the indifference map will be useful and Pareto's discovery is a great one.

Utility as a Standard of Measurement

In discussing the equilibrium of tastes and obstacles Pareto achieved a high degree of objectivity. By using the indifference scale as a given hypothesis, he erected a theory of equilibrium essentially independent of the utility concept. But the independence is not as complete as one would like. In his discussion of tastes he again falls back on the old concept. Here he considered the problem of the consumption of one good in relation to others.[2] This involved a subjective valuation or more precisely a comparison of utilities of different goods for the same person. What he intended to do was

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[1]An excellent discussion on the indeterminateness of the utility function is to be found in the article by J. R. Hicks, "A Reconsideration of the Theory of Value," Economica, February, 1934, pp. 52-55.

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[1]An excellent discussion on the interrelatedness of the utility function is to be found in the article by J. R. Hicks, "A Reconsideration of the Theory of Value," *Economica*, February, 1934, pp. 52-56.

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to set up this comparison as an objective index of human choice. Utility would still be a function of quantity only, and after observation of the fluctuation in quantities exchanged, the data so attained was to be used objectively to explain tastes. The theory was aimed at establishing the results of human choice in terms of quantities exchanged and the ratios of such quantities (prices).

For this purpose Pareto made two basic assumptions.[1] He assumed first that though utility is not measurable, the individual is capable of knowing whether the utility derived from one combination of goods equals, is greater, or is less than that of another. In terms of choice the individual knows whether or not he prefers one combination to another or is indifferent as to the choice of either. The second assumption is that the individual knows whether the change in utility due to transition from one combination to another is equal to, greater, or less than the change from this latter combination to a third. Under the first assumption utility is an index only, as we have explained above, and the theory remains objective. The second assumption, however, while an outgrowth of the first, is no longer purely objective, because it involves a type of empirical observation which is largely impossible. Such results cannot be had from observation only.

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[1]Pareto, Manuel d'Economie Politique, op. cit., pp. 264-265.

They rely on psychological introspection and therefore fall outside the realm of objective human choice.

This is further brought out in the actual argument. So long as he discusses total ophelimite and its use as an index he remains under the first assumption. But when he finds it necessary to discuss elementary ophélimité (marginal utility) and its index, he falls under the second assumption. This is particularly true with regard to his definitions of complementary and competitive goods. These he defines in terms of the variations in the ratios of the marginal utilities of the goods involved.[1] While it is possible to observe whether one combination is preferred to another, it is quite a different thing to observe the changes from one transition to another, and what is more--to assume that they can be taken as an objective index of human choice. Let us assume for example three combinations, 1, 2, 3, with the utility of 3 greater than that of 2, and 2 greater than that of 1. If our theory is to make any sense under the first basic assumption, the individual must always choose 3 in preference to 1 or 2 regardless of what his gain is from 1 to 2 or 2 to 3. In terms of acts of choice it is meaningless to say that he prefers the transition from 2 to 3 in preference to that of 1 to 2, for he will always choose 2 to 3 even if the gain in utility is less than the gain from

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[1]<u>Ibid</u>., pp. 251-252; pp. 265-268.

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[1] Ibid., pp. 281-282; pp. 282-283.

the transition from 1 to 2. In order to compare the gains then, the individual must fall back on psychological introspection so that the second underlying basic assumption is not capable of expression objectively in terms of acts of choice.[1] If this assumption is used together with the first one then it must follow that utility is measurable.

But if utility is measurable and a theory is built upon that measurability, then it cannot be an objective theory. The only way to keep it objective is to state it in terms of the first assumption only. Chapter 3 of the Manuel d'Economie Politique is for the most part stated in such terms, but Chapter 4 makes use of the marginal utility concept and the law of diminishing marginal utility. The discussion of related goods in particular is framed on fluctuations of "ophélimité élementaire." Pareto's theory then to be objective would have to be restated in terms of the first assumption only, which would do away with the measurability of utility.

That utility is immeasurable seems not to have been recognized by Pareto for he always speaks of it quantitatively. At any rate, if he did recognize it, he did not restate his theory

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[1]Lange, "The Determinateness of the Utility Function," op. cit., pp. 222-223. Lange shows that by starting with an arbitrary measure of utility and a fixed unit of transition it becomes possible to state how many times one combination is preferred to another. On the indifference map this would mean that the indices take on the meaning of by how much one curve is greater than another. This is not a measurement of total utility but it shows how the second assumption lends itself to partial measurement.

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minus the quantitative aspects.[1] Hicks suggests that he did not recognize this until the late stages of his work in economics and he therefore continued to use the concepts derived from earlier ideas. Furthermore when he did recognize it, he did not take the trouble to rework his earlier conclusions.[2] It would seem then that if Pareto did recognize this, he missed a great opportunity. However, this opportunity was finally taken up by Dr. Hicks and Mr. Allen. The work of these two culminated in the volume, Value and Capital, by Hicks, which remains today the latest constructive work on the subject. This is in the beginning a restatement of Pareto without the quantitative aspects and it allows for some quite remarkable developments.

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[1]Another work of Pareto appeared in 1911, "Économie Mathématique," Encyclopédie des Sciences Mathématiques, Vol. I, in which he restates the argument of the Manuel d'Economie Politique with minor change.

[2]J. R. Hicks, Value and Capital (London: Oxford University Press, 1939), p. 19.

CHAPTER III

FINAL ABANDONMENT OF THE UTILITY CONCEPT

Pre-Hicksian Development

Hicks was one of the few people to see the possibilities inherent in the great discovery of Pareto, and to take advantage of the new directions which Pareto's theory offered. Once he decided to do away with quantitative utility, his task was laid out. He had to review the theory of Pareto, adjust and reconstruct the basic concepts tainted in any manner with quantitative utilitarianism and then work out the results, regardless of where they led. His burden was eased to some extent by the work of W. E. Johnson and more directly by the work of R. G. D. Allen. Johnson's work apparently did not spring from Pareto, but was rather an outgrowth of Edgeworth. He proceeded from Edgeworth's discussion of barter between two individuals. Here as we said previously, the constant utility curves are constructed so that the coordinates of any point represent two variable quantities, one of which is acquired and the other is sacrificed for the former. The acquisition yields utility, the sacrifice disutility; the net utility thus increases with the decrease of sacrifice

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or the increase of acquisition.[1] The position of equilibrium is still relatively indeterminate. Johnson then proceeded to alter Edgeworth's curves by turning them upside down. In effect he transforms Edgeworth's curves into the customary indifference curves as we know them. He was thereby enabled to deal with those cases in which two quantities contribute positively to the resulting utility and also by analogy to the case where numerous variables contribute to utility. More important, however, is the fact that he was able to treat the case where the income of the consumer was fixed. It will be remembered that this was also treated by Fisher, but Johnson's work in this respect is much advanced over that of Fisher and is particularly to be noted because it had a great influence on Hicks, especially in the latter's development of the income and substitution effects. The technique used will not be discussed at this point, but we shall point out the influence of Johnson when we discuss these effects below.

Johnson's work was taken up by Allen and it served to direct Hicks' interest to the problem.[2] There followed a period of close collaboration between Allen and Hicks, the

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[1]W. E. Johnson, "Pure Theory of Utility Curves," Economic Journal, December, 1913, p. 484.

[2]R. G. D. Allen's first work appears in "Nachfragefunctionen für Güter mit korrelierten Nutzen," Zeitschrift für Nationalokomie, March, 1934. Also "A Comparison between Different Definitions of Complementary and Competitive Goods," Econometrica, April, 1934.

results of which appear in "A Reconsideration of the Theory of Value," Economica, 1934.[1] This collaboration covered a period of almost five years and was so close that it has been completely impossible to separate the contributions of each. It finally culminated in the volume, Value and Capital, by Hicks, in which he restates this earlier work and then proceeds on his own to develop from it a theory of dynamics.[2]

An interesting feature of this entire development is the fact that it was completely anticipated by a Russian economist, E. Slutsky of the University of Charkov. Subsequent to the publication of their findings in Economica, Allen and Hicks came across the work of Slutsky which had been published in Italian in 1915.[3] Slutsky set out essentially the same material, but his work is highly mathematical and contains very little discussion about the significance of the theory. It is for these reasons, and also probably because the work was published in Italian at a time when Italian economists paid little attention to the problem, that his work remained unnoticed for so

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[1]R. G. D. Allen and J. R. Hicks, "A Reconsideration of the Theory of Value," Economica, February and May, 1934. The first part by Hicks sets out the new theory in words; the second part by Allen discusses it mathematically.

[2]Hicks, Value and Capital, op. cit.

[3]E. Slutsky, "Sulla teoria del bilancio del consumatore," Giornale degli Economisti, July, 1915. See also R. G. D. Allen, "Professor Slutsky's Theory of Consumer's Choice," Review of Economic Studies, February, 1934.

long and had to be rediscovered. Nevertheless it is apparent now that Slutsky saw the shortcomings of Pareto and set out in the new directions indicated. The following paragraphs will be devoted to the territory opened up by Slutsky and so ably developed by Allen and Hicks.

The New Concepts

The new theory of Allen and Hicks involved a thorough purge of the utility theory. All concepts tainted by quantitative utility were discarded and replaced where needed by new ones without the objectionable implications.

1. The first concept to go was the concept of marginal utility, for if total utility quantitatively defies definition, then so does marginal utility; and it is significant that the new theory of value to be constructed does not require a precise definition of marginal utility. It will still be possible under the new system to give meaning to the ratio of two marginal utilities when quantities of each commodity are given. This is represented by the slope of the indifference curve. However, this is merely of passing interest and independent of the new concept. If marginal utility is done away with, what then is needed in its place? Some concept which will enable us to compare adjacent combinations of equivalent value. (We still do not say what or how much is this value.) Hicks calls this the "Marginal Rate of

Substitution." The marginal rate of substitution of X for Y is the quantity of y which would just compensate the consumer for the loss of a marginal unit of x or the quantity of x which would just compensate the consumer for the loss of a marginal unit of y.[1] If the consumer is moving down the curve and does not get the proper amount of x, he will be worse off than before the substitution took place. If he is moving up the curve and does not get the proper amount of y, then he will also be worse off. It should be seen that this is merely a rate of substitution which enables us to express that quantity which will leave the consumer, after substitution, exactly as well off as before. The essential character of the concept is one of substitution. It is entirely independent of any quantitative measure of utility.

If an individual is to be at equilibrium with respect to a given system of prices, his marginal rate of substitution between any two goods must equal the ratio of their prices,

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[1]There is a change in definition between the article in Economica and the text of Value and Capital. The above definition is that given in Value and Capital. In Economica the definition embraces a different point of view. The amount of y necessary to offset the loss of a marginal unit of x is defined as the marginal rate of substitution of Y for X. At first glance it appears that the reason for the change lies in the footnote on page 20 of Value and Capital where Hicks explains why he went from increasing to decreasing marginal rate of substitution. On reflection, however, it can be seen that it is possible even on the basis of the definition of the marginal rate of substitution of Y for X to have a decreasing rate. The real reason for the change is merely that Hicks wanted to make his terminology similar to that of Marshall.

otherwise there would be an advantage in substituting some quantity of one for an equal value (at the market rate) of the other. This is the form in which we now write the equilibrium condition of the market. (Diagram 7)

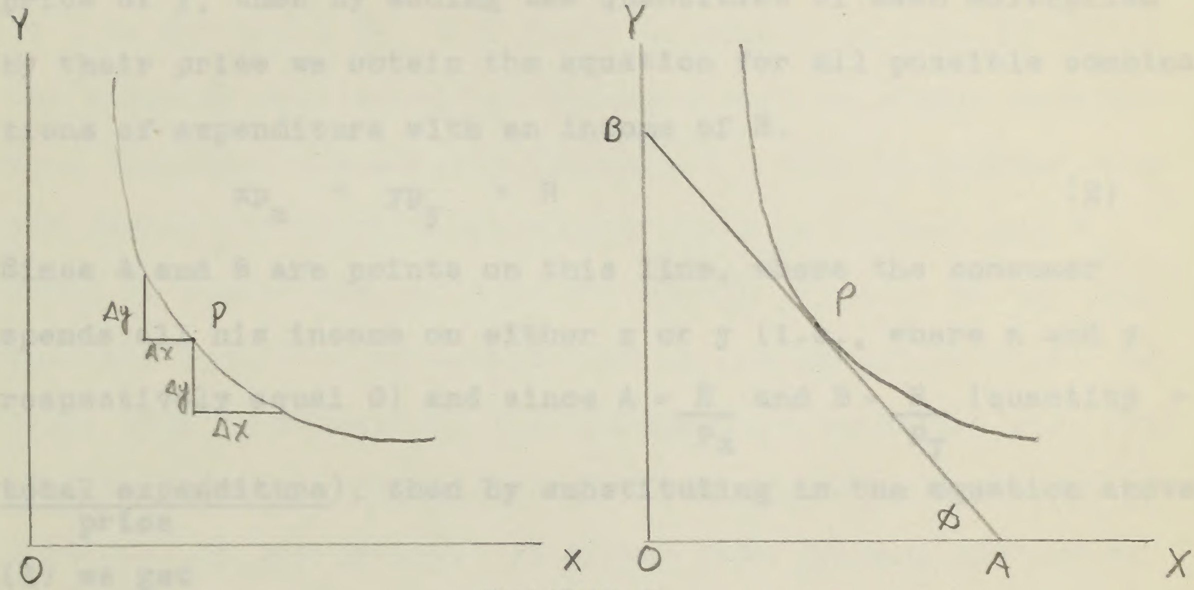

DIAGRAM 7

It will help to examine this in simple mathematical terms. The marginal rate of substitution between X and Y may be designated as S_{xy}. It is the ratio of the increments at point P, $\frac{\Delta y}{\Delta x}$.[1] From the theory of limits it follows that the ratio of increments at point P is measured by the slope of the tangent line at that point, that is

$$S_{xy} = \text{slope of line AB which} = \tan \phi = \frac{OB}{OA}$$

The equation of a straight line which cuts OX at A and OY

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[1]It should be noted that Δy is positive and Δx is negative. To avoid confusion the ratio is defined as $-\frac{\Delta y}{\Delta x}$ so it is always positive.

at B is

$$\frac{x}{A} + \frac{y}{B} = 1 \qquad (1)$$

If we let R = total income, p_x = the price of x and p_y = the price of y, then by adding the quantities of each multiplied by their price we obtain the equation for all possible combinations of expenditure with an income of R.

$$xp_x + yp_y = R \qquad (2)$$

Since A and B are points on this line, where the consumer spends all his income on either x or y (i.e., where x and y respectively equal 0) and since $A = \frac{R}{p_x}$ and $B = \frac{R}{p_y}$ (quantity = $\frac{\text{total expenditure}}{\text{price}}$), then by substituting in the equation above (1) we get

$$\frac{x}{\frac{R}{p_x}} + \frac{y}{\frac{R}{p_y}} = 1 \text{ or } xp_x + yp_y = R \qquad (3)$$

At any point then on the price line AB the slope $= \frac{OB}{OA}$. At P where AB is tangent to the indifference curve we have already shown that the ratio of increments is measured by $\frac{OB}{OA}$. Now again substituting the values OB and OA, we get

$$\frac{OB}{OA} = \frac{R/py}{R/px} = \frac{p_x}{p_y} = S_{xy} \qquad (4)$$

or as we said above the condition of equilibrium is that the marginal rate of substitution is proportional to price. This

will appear even more rational if we understand that the term S_{xy} is the measure of the rate of conversion of x into y and is independent of price. The price, on the other hand, measures the rate of exchange of the goods against each other. If price of x is $10 and y $2, then ratio of prices is 5 to 1. If in the exchange the individual values a unit of y more relative to x than does the market, then his S_{xy} will be greater than the market and he will exchange y for x and reduce his S_{xy} until it equals the ratio of the market prices. We see now that the condition of equilibrium on the market is where the price line is tangent to the indifference curve and also that at this point the quantities of x and y purchased will be proportional to the prices. In effect this is very similar to Marshall. The marginal rate of substitution of X for Y is what Marshall would call the marginal utility of X in terms of Y.[1] He also showed that this was proportional to the prices. The results so far are very similar.

2. The second concept to go is the principle of diminishing marginal utility. The reason it is discarded is the same reason for which the principle of marginal utility was discarded, for if marginal utility has no exact meaning, except in a subjective sense, then certainly diminishing marginal utility also has no exact meaning. It must therefore give way

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[1]See footnote, page 47.

to a new principle, that of decreasing marginal rate of substitution. The new principle may be stated as follows. If we start with a given quantity of x and y, and then increase the amount of x and decrease the amount of y in such a way that the consumer is neither better nor worse off after the exchanges, then the amount of y which has to be subtracted in order to set off the second exchange will be less than that which has to be subtracted in order to set off the first exchange. In other words the more we substitute y for x, the less will be the marginal rate of substitution of x for y (see Diagram 7). In times of our previous analysis the more we substitute y for x the less will be $\frac{\Delta y}{\Delta x}$, S_{xy}. This condition is expressed on the indifference diagram by drawing the indifference curves convex to the axes.

The principle of diminishing marginal utility and the principle of diminishing marginal rate of substitution are not the same thing, or more precisely diminishing marginal utility and convexity of the indifference curves are not the same. The increase in x may affect not only the marginal utility of x, it may also affect the marginal utility of y, and when we consider the effect of one on the other it is quite possible to have cross effects cancelling each other. For example, it is quite possible for an increase in x to diminish the marginal utility of y which in turn increases

the marginal utility of x. The cross effects then may cancel out without any change in the marginal utility of x, or it is even plausible that a movement along the indifference curve to the right will actually increase the slope of the curve. This is no doubt an unusual case, but it is entirely possible under the principle of diminishing marginal utility.

But Hicks needed an assumption which would entirely preclude any such possibility, and by assuming the diminishing marginal rate of substitution he accomplished just that. The replacement therefore is not a mere change in terminology; it is a "change in the basic foundation of the theory" and as such requires a definite justification. The justification is as follows. Hicks needed the principle of diminishing marginal rate of substitution for the same reason that Marshall needed the principle of diminishing marginal utility, for unless at the point of equilibrium, the marginal rate of substitution is diminishing, there will be no stable equilibrium. Even if the marginal rate of substitution equals the price ratio, so that no advantage accrues in the acquisition or sale of a marginal unit of x, nevertheless if marginal rate of substitution is increasing there will be advantage in acquiring a larger quantity, because it will be possible to proceed to an indifference curve of a higher index. This is explained in Diagram

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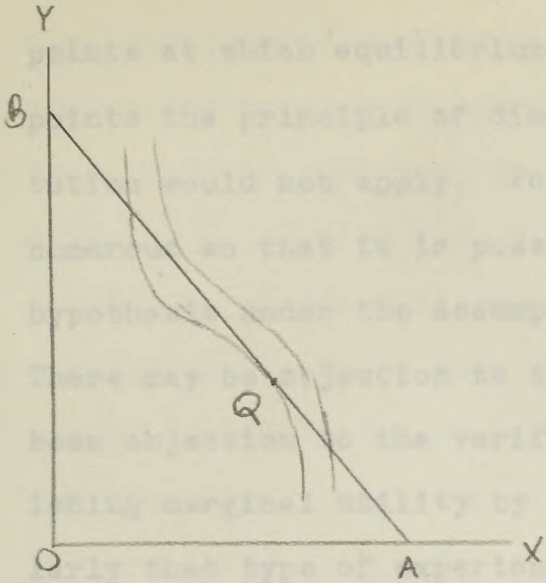

DIAGRAM 8

8.[1]

At point Q on the diagram the marginal rate of substitution equals the price ratio, i.e., at this point the indifference curve is tangent to the price line. However, if the individual moves away from point Q, he will arrive at an indifference curve of a higher index. It is clear then that unless at point Q the marginal rate of substitution is decreasing, or in other words, the indifference curve is convex to the axes, there will be definite advantage to move away from point Q so that this will not be a point of stable equilibrium. Actually on price line AB, Q is a point of minimum, not maximum utility. It is clear therefore that, under a given set of prices, for any point to be a point of equilibrium, the marginal rate of substitution at that point must be decreasing. The use of this principle as a basic assumption means simply that in a given region any point with appropriate prices may be a point of equilibrium. However, there are conceivably

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[1]In Value and Capital, page 21, Figure 4, Hicks' diagram is drawn so that a part of the curve has an upward slope to the right. This implies negative utility and is not what was intended. The curve should be falling throughout.

points at which equilibrium may not take place.[1] At these points the principle of diminishing marginal rate of substitution would not apply. Fortunately these points are not numerous so that it is possible to use the principle as a hypothesis under the assumption that it is universally true. There may be objection to this assumption just as there has been objection to the verification of the principle of diminishing marginal utility by the appeal to experience, particularly that type of experience which offers no opportunity for testing. But this objection can be overcome largely because the concept itself is objective. The purpose of the principle is to deduce laws of market conduct, which deal with the reactions of consumers to changes in market conditions. Certainly when conditions change we have observed that the consumers take up new positions of equilibrium. Unless at each of these positions the marginal rate of substitution is diminishing, equilibrium will be impossible. Then to make the hypothesis universal all we need to do is make a further assumption that the condition will prevail at all intermediate points between

[1]Such points would take the form of blind spots on the indifference diagram within which no stable equilibrium is possible. Hicks includes here the possibility of the case of Buridan's ass, where the consumer with a given income, confronted with a given set of prices, is unable to decide between a number of different expenditure possibilities. But even here, he says, the consumer does not hesitate indefinitely. He finally decides to buy some quantities so that the principle of marginal rate of substitution must sometimes be true.

two positions of equilibrium so that there are no kinks in the curves between the positions. If there are kinks then there will be some systems of prices at which the consumer will not be able to make his choice of expenditure. But people do not hesitate indefinitely so that there are points of equilibrium where choice is made. The principle of diminishing marginal rate of substitution then merely rules out the temporarily peculiar situation.[1] The continuous indifference curve with downward and diminishing slope appears to be the simplest assumption possible "and in fact its accordance with experience seems definitely good."[2]

We have now disposed of those concepts from the utility analysis which are objectionable and we have replaced them with concepts more suitable for the purpose. The mere replacement of these opens up avenues for which additional concepts are required. These we shall take up below. But before we proceed an additional qualification is necessary. Until now the new theory has been simplified by the condition that consumer's choice is restricted to the expenditure of two goods

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[1]The principle of diminishing marginal rate of substitution rules out only certain kinks in an otherwise irregular indifference curve--those kinks which would result in upward sloping stretches which are concave to the origin. Kinks which are merely sharp bends or corners which make for two or more tangents at the same point are allowed so long as they are convex to the origin, i.e., so long as the tangents are negatively sloped.

[2]Hicks, Value and Capital, op. cit., p. 24.

only. This simplification can now be removed so that the case may be considered where expenditure occurs between more than two goods.

For expenditure divided among three goods it is still possible to use the indifference program, but this would have to be done in three dimensions and would therefore become much more involved. For more than three goods geometry no longer is a tool since the necessary dimensions are lacking. This, however, does not substantially affect the principles previously discussed, for if we now make the provision that the quantities consumed of all other goods remain the same, the marginal rate of substitution can still be defined as before, and, as before, the consumer will still be at equilibrium if the marginal rate of substitution between any two goods equals the ratio of their prices. The condition with regard to diminishing marginal rate of substitution is somewhat more complicated. In order that stable equilibrium be possible among several commodities, it is necessary that expenditure be made so that no possible substitution of equal market values bring the consumer to a more advantageous position. This means that not only between any two goods must the marginal rate of substitution be diminishing but also that potential complicated substitution of other goods be ruled out. Hicks expresses this by saying that the marginal rate of substitution must diminish for substitutions in every direction.

If any two possible goods were to be isolated the marginal rate of substitution between them would be diminishing; thus this provision eliminates those potentially complicated substitutions. The justification for this is similar to that given in the simpler case.

Income and Substitution Effects

The fundamental concepts developed above have been based entirely on the substitution relationship, but substitution itself is inadequate to explain all types of market behavior. It refers only to a particular change, that which occurs when one commodity is replaced by another; in our terms it refers to the case where an individual moves from one position to another on the same indifference curve. This, however, is not the only change which may take place. In response to new conditions the individual does not necessarily move along the same indifference curve. More often a change in market conditions will make him either better or worse off so that instead he moves to a higher or lower indifference curve. It is evident then that the new information required is not about the shape of a particular curve but rather about the relationships between the many curves.

Two changes effecting these relationships may be designated. A change in price may bring about increased substitution of one good for another or it may act like an increase

in income. It is therefore convenient to designate these changes as the income effect and the substitution effect. The income effect will be considered first, since it is easier to deal with the substitution effects after the income relationships are understood.

Referring now to the indifference diagram, let us continue to assume the prices of X and Y as given, but we shall make the new proviso that income is to vary. AB will be the income line and the point of equilibrium will be located at point P where AB is tangent to the indifference curve. When income varies AB will move to the right or to the left. If prices remain the same and income increases, then the new income line will be A'B' which is parallel to AB. The new point of equilibrium will be at P' where A'B' touches another indifference curve. As income continues to increase the points of equilibrium trace out a curve CD which indicates the new consumption combinations which result from our given change. Hicks calls this curve the income-consumption curve. (In <u>Economica</u> he called it the expenditure curve. This clearly was a poor name.) In effect it is Pareto's "line of exchanges," but

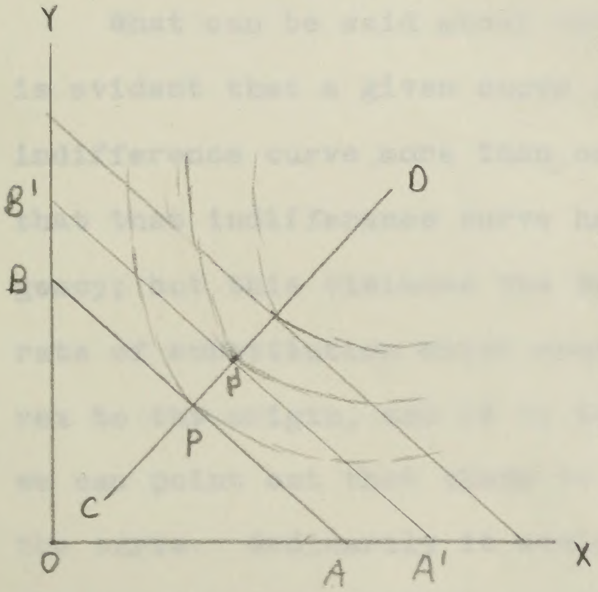

DIAGRAM 9

its present significance is entirely different.[1] Pareto's device attempted to show the point of equilibrium between two opposing indifference systems. The curve as it is used by Hicks merely shows the relationships between one curve and another of the same indifference system. Geometrically the curves are the same, but as a tool Hicks' use is far more effective.

What can be said about the form of this curve? First it is evident that a given curve cannot intersect a particular indifference curve more than once. If it did this would mean that that indifference curve had more than one point of tangency; but this violates the assumption of diminishing marginal rate of substitution which makes the indifference curves convex to the origin, and it is therefore impossible. Secondly we can point out that there is no limitation to the slope of the curve. Ordinarily it would slope upward to the right, but depending on conditions, it is quite possible for it to turn to the left or downwards to the right. If the income-consumption curve is positively inclined,this indicates that an increase in income will increase the consumption of both commodities, X and Y. If it is negatively sloped, an increase in income will increase the consumption X but decrease that of Y. It also may be backward sloping to the left; indeed it can take almost any shape without cutting an indifference curve more than once. This is clearly seen in the case of

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inferior goods where the curve is negatively sloped. It is found whenever one commodity X is an "inferior" good largely consumed at low levels of income, but replaced, or partially so, by goods of a higher quality when income increases. Margarine is one example, but there are many others; indeed most of the poorer qualities of goods are in this sense inferior goods.

Although the apparatus used above is valid only for the case of two goods X and Y, it is also valid no matter how many are the goods among which income is distributed. If income increases and is spent, then there must be an increase in consumption in some direction if not in all. The increase may be limited to a few goods; some may even diminish, but this is of no consequence since the increase must be felt in some direction.

This stage of the discussion affords an interesting comparison with the theory of Marshall where he deduces the downward slope of the demand curve from the law of diminishing marginal utility. To do this he had to assume that the marginal utility of money is constant. Therefore the ratio between the marginal utility of a commodity and its price is constant; in other words there is no distinction between diminishing marginal utility and diminishing marginal rate of substitution of that commodity for money. If price falls, marginal utility must also fall, but this also implies an

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increase in the amount demanded. It would appear then that constant marginal utility meant for Marshall that changes in the consumer's supply of money would not affect the marginal rate of substitution between money and a particular commodity. If we let y equal money, then an increase in income will not affect the consumer's S_{xy} except as it is affected by changes in (the marginal utility of) x alone. The demand for x therefore becomes independent of income, and this is precisely what the constant marginal utility of money did for Marshall. It allowed him to disregard income effects. For his purposes the device was practically harmless, but it does not give a thorough theory showing all the relationships. One of the chief advantages of the indifference tool is, as we shall see, that it allows a clear explanation of many of the numerous relationships between demand, income, and price.

Let us now look at effect of change in price aside from the income effect. Beginning with two goods X and Y, the conditions now are that income remains the same and also the price of Y. The price of X however will be allowed to vary. This situation is illustrated on Diagram 10. Here consumption possibilities are indicated by the expenditure lines AB and A'B depending on how the price of X varies. Since OB is income measured in terms of Y and is constant, then regardless of how the price of X changes the total expenditures will remain the same. As the price of X falls the amount consumed

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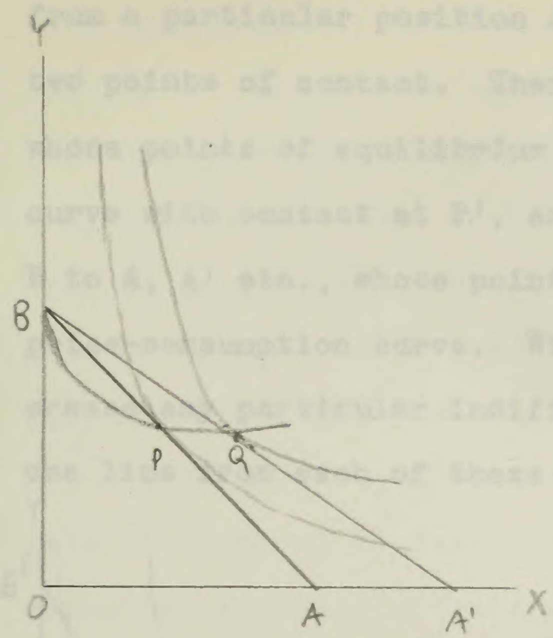

DIAGRAM 10

will increase. The expenditure line will move from AB to A'B. The point of equilibrium where the expenditure line is tangent to the indifference curve will move P to Q. The curve BPQ which is traced out by the changing points of equilibrium will be called the price-consumption curve. It shows how consumption varies, when the price of X changes, all other things remaining the same. In effect if Y is money it is a demand curve for X.[1]

Here also it follows that the price-consumption curve can intersect a particular indifference curve only once and also that there is no limitation on the shape of the curve.

Having developed the two curves, let us now coordinate them to examine the effects of a fall in price.[2] Starting

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[1]If Y measures money then the relationship between the price-consumption curve and the traditional Marshallian demand curve is as follows: the latter shows price per unit along the ordinate, the former total expenditure. A price-consumption curve rising from left to right corresponds to a demand curve greater than unity. A price-consumption curve which declines from left to right will correspond to an inelastic demand curve.

[2]In Economica Hicks develops the concepts of income-elasticity and elasticity of substitution and he frames his discussion in these terms. In Value and Capital he drops them since he saw that they were not necessary at this point to explain the two effects. For the sake of clarity I follow Hicks in this respect. For further reference to these concepts see also Hicks, Théorie Mathématique de la Valeur (editor, Georges Lutfalla; Paris: Actualités Scientifiques et Industrielles, 1937).

from a particular position AB we get two sets of lines and two points of contact. There are the lines parallel to AB whose points of equilibrium trace out the income-consumption curve with contact at P', and there are the lines passing from B to A, A' etc., whose points of equilibrium trace out the price-consumption curve. Within the limits of the price decrease any particular indifference curve must be touched by one line from each of these sets. Thus in Diagram 11 indif-

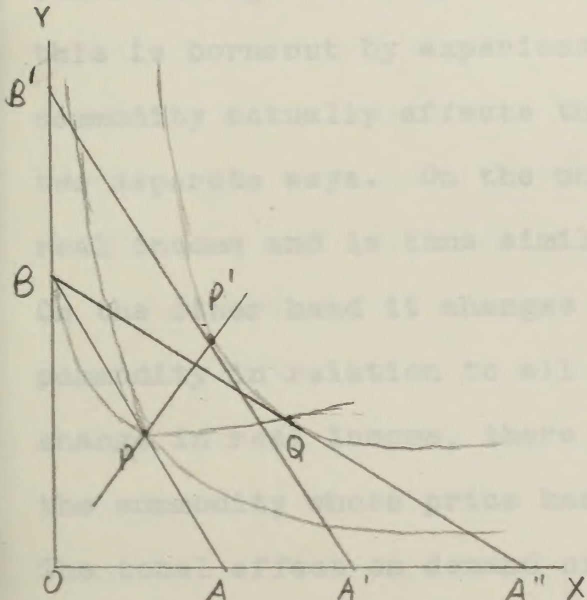

DIAGRAM 11

ference curve number 2 is touched by a line parallel to AB at P' and also by a line passing through B at Q. P' and Q are the points of equilibrium. Now from the fact that curve 2 is higher than curve 1, and from the convexity of the indifference curves it must follow that point Q always lies to the right of P'. Since we are varying the price of X only, it likewise always follows that as we proceed to higher indifference curves the price-consumption line will always be to the right of the income-consumption line through P. What does this all mean in terms of consumer consumption? The significance of this can be seen if we compare the directions of movement. When the price of X falls the consumer moves along the price consumption

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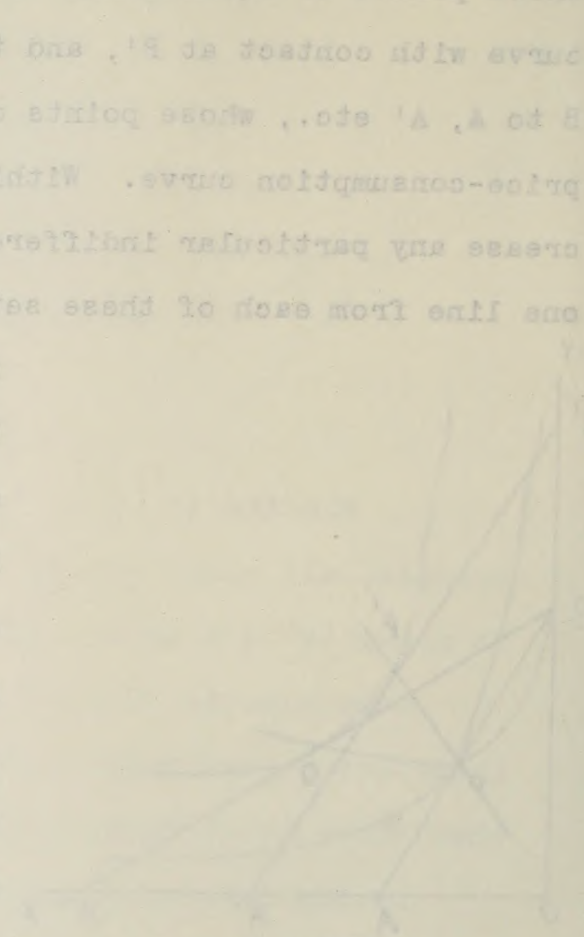

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line from P to Q. But the drop in the price of X is equivalent to an increase in income so that the movement from P to Q is the same as movement from P to P' along the income-consumption curve. This in turn is equivalent to a movement from P' to Q on indifference curve 2. The movement from P to Q involves then two steps, one directly along the price-consumption line, this we call the substitution effect, and the other through P' to Q, this we call the income effect. And this is borneout by experience, for a fall in the price of a commodity actually affects the demand for that commodity in two separate ways. On the one hand it raises the consumer's real income and is thus similar to an increase in money income. On the other hand it changes the relative prices of a given commodity in relation to all others, so that aside from the change in real income, there will be a tendency to substitute the commodity whose price has dropped for other commodities. The total effect on demand of a drop in price can then be thought of as the sum of these two effects, the income effect and the substitution effect.

The relative importance of each of these effects will depend on the proportion of total income spent on one good X and all other goods Y. The extent by which the consumer is made better off by a fall in the price of X thus depends on the proportionate amount of X which he was originally buying to his total income. The greater this proportion, the greater

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will be the increase in real income and the more important will be the income effect. But if the proportion of X is small, then the gain to the consumer will be small and whatever gain there is, is more than likely to be taken up by substituting more X. Here we say the income effect is swamped by the substitution effect. It is on the basis of this last point that Marshall was partially justified in assuming the constant marginal utility of money. He did not work it out in just this manner but for those cases where the substitution effect does swamp the income effect his theory will apply. However as we have previously pointed out, it does not tell the whole story.

Furthermore, it should be pointed out that there is a great difference in the reliability of these effects. This in a certain measure also substantiates Marshall, for we can definitely say that the substitution effect is absolutely certain. From the original assumption of the principle of diminishing marginal rate of substitution it must follow that when the price of a commodity falls the substitution effect will work for an increase in the demand for this commodity. Here Marshall is on safe ground. But with the income effect this certainty is not so absolute. Ordinarily through the income effect a fall in the price of a commodity will bring about an increase in the amount demanded, but this will not always be the result. The income effect is

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not always reliable. This is particularly true in the case of inferior goods, where a fall in price leads rather to a decrease in the amount purchased.

This latter case is worth noting. An inferior good is defined as one consumed at low income levels, but which is replaced or partially so by other goods when income rises. This is particularly well brought out by the direction of the income-consumption line. In Diagram 12 it will be noted that this line at first rises with the increase in income, but as income continues to increase it bends around backwards until it is negatively sloped. In this case Y is the inferior good consumed at low levels of income. As income increases there will be an increase of both X and Y until a certain point M beyond which the income effect with respect to Y will no longer be felt. On the contrary further increases of income bring about a decrease in the consumption of Y causing the income-consumption curve to be negatively sloped. The resulting effect may be considered as a negative income effect. What happens is that beyond point M the consumer would rather buy other goods which he considers of a higher order so that the increase

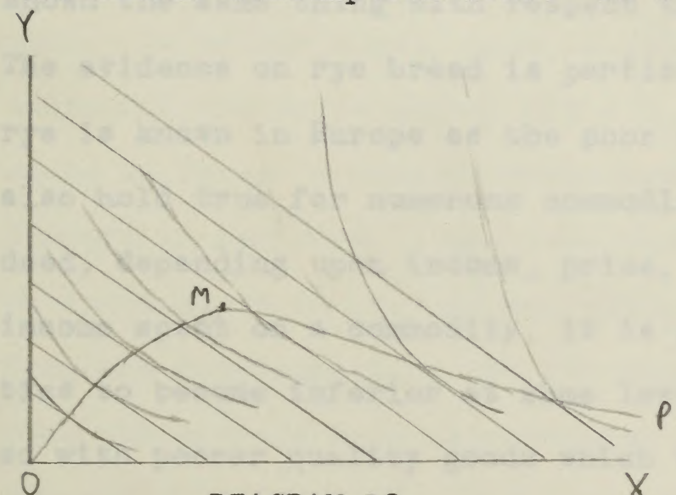

DIAGRAM 12

in income converts Y into a nuisance. Therefore instead of increasing his consumption of Y he substitutes other goods in its place. It is for this reason that the income effect is not always reliable.

Moreover this has been well brought out by statistical study. Allen and Bowley have shown this to be true for margarine which is obviously a case in point.[1] They have also shown the same thing with respect to rye bread and flour.[2] The evidence on rye bread is particularly interesting because rye is known in Europe as the poor man's crop. The same will also hold true for numerous commodities other than food; indeed, depending upon income, price, and the proportion of income spent on a commodity, it is possible for most commodities to become inferior at some level.[3] This is especially so with poorer quality goods which become inferior with very moderate increases of income and are rapidly substituted for

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[1]R. G. D. Allen and A. L. Bowley, Family Expenditure (London: P. S. King and Son, Ltd., 1935), p. 41

[2]Ibid., p. 29. Notice particularly the negative sloping line for rye bread. The problem is again treated theoretically in Chapter 8. Note especially their figure 4, page 106, which corresponds essentially to my Diagram 12. See also W. E. Johnson, "Pure Theory of Utility Curves," Economic Journal, December, 1913, pp. 491-492, for a similar discussion.

[3]Inferior goods are always found at low levels of income so that it may appear that it is the income and not the good which is inferior. Actually the negative income effect and the low level income are both contributing factors.

goods of a higher quality. This is further basis for the unreliability of the income effect.

In the previous paragraphs we have set out the development of the income and substitution effects as it stands today. This of course is largely attributable to the work of Allen and Hicks. Their work, however, is very much the outgrowth of groundwork laid by Johnson; in fact the geometric technique is practically the same. Johnson develops the same curves as we saw above, only he calls them the expenditure curve and the price curve. In all probability he was influenced by Pareto's line of exchanges, but the outstanding feature in his work is the fact that he treats the effect of an increase in income and the effect of a fall in price.[1] From the development of the two new curves he was able to see that the effects were interrelated and he proceeded to work out the relationships. Moreover, in spite of the fact that his work is based on the utility concept he saw that the assumption of constant marginal of money was not absolutely necessary to work out the theory. In this respect he surpasses Marshall. Yet in spite of his innovations he still

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[1] Johnson, op. cit. The development of the price and expenditure curves is contained in the above article. The work is largely mathematical but there is enough verbal explanation so that one can see how he approaches the two effects. Also of great interest is the fact that he treats the problem of inferior goods although he does not call it by that name, and also consumer's surplus which we shall examine below.

clung to the idea of utility. It is for this reason that we refer to his work as groundwork. However,it was a thorough type of groundwork, an intermediate step so to speak, in the development of indifference analysis and one which is apt to be overlooked. But that it determined the direction of the later development can hardly be doubted.

Before leaving the income and substitution effects it will be well if we set them out in non-theoretical terms so that there will be no misunderstanding in the meaning. As we have pointed out above the effect of price on consumption falls into two effects. A drop in price will increase the buying power of the consumer and a portion of this increase may be devoted to increased consumption of the cheapened commodity (the income effect) or it may induce the consumer to substitute more of the cheapened commodity for other goods (the substitution effect). In the graphic description we started with an increase in income and showed the same increase in satisfaction which would have been possible through a fall in price. In the example below for the sake of clarity we will start from the other end.[1] Let us assume a family expenditure on food distributed so that over a given period $39 is spent. There will be

[1]This numerical example is based on the article by Fritz Machlup, "Professor Hicks' Statics," Quarterly Journal of Economics, February, 1940, pp. 279-282.

30 lbs. of meat	@	.50 per lb......	$15.00
60 baskets other food	@	.40 per basket..	24.00
			$39.00

If the price of meat falls to .30 cents per pound the same combination can be bought for $33, and $6 will be released for the purchase of other goods. Let us set aside the $6 released for the moment. What will be the effect of the drop in price of meat? The drop in price will induce the consumer to substitute meat for the other food so that spending $33 he will buy

34 lbs. of meat	@	.30 per lb......	$10.20
57 baskets of other food	@	.40 per basket..	22.80
			$33.00

Without using the $6 the consumer substituted 4 pounds of meat for 3 baskets of food. Now if we allow this $6 to be used so as to feel the full effect of the fall in price, it will be possible for the consumer to purchase more of both items. Consumption will appear as follows:

42 lbs. of meat	@	.30 per lb......	$12.60
66 baskets of other food	@	.40 per basket..	26.40
			$39.00

The effects of the drop in price in meat may be summed up as follows:

Original consumption	30 lbs.
Increase due to substitution effect 4 lbs.	
Increase due to income effect 8 lbs.	
Total effect of fall in price	12 lbs.
Meat consumption at lower price	42 lbs.

The fall in the price of meat, however, not only affects the consumption of meat but also the consumption of other foods. This may be summed up as follows:

Original consumption	60 baskets
Decrease due to income effect on meat	-3 baskets
Increase due to substitution effect of meat	+9 baskets
Total effect of fall of price of meat on consumption of other foods	+6
Consumption of other foods at lower price of meat	66

The intermediate situation in the above example is purely imaginary, because ordinarily the effect of the $6 will be felt immediately. However it gives us a clearer picture of the whole process. The final situation shows an improvement worth $6 over the imaginary intermediate position. The intermediate position itself is an improvement over the original position, but by exactly how much cannot be determined. We do know that the consumer preferred 34 lbs. of meat and 57 baskets of food to 30 lbs. of meat and 60 baskets of food which he would have bought for exactly $33. The superiority

of the intermediate position then is more than \$6.

But the superiority of the final position over the intermediate is exactly \$6 so that we can say that the superiority of the final position over the original must be over \$6. At unchanged prices the consumer would have spent \$47.40 for the quantities in the final position, or \$8.40 more. This does not correspond to the increase in satisfaction,for if the consumer had originally \$47.40 to spend he would have distributed it differently, but it enables us to set limits to the income effect. We can therefore conclude that the income effect of the drop in price of meat is equivalent to an increase in income of more than \$6, but less than \$8.40. The income effect then is in excess of the money income released through a fall in price. To analyze the total effect it would be necessary to know the indifference map of the individual concerned. This of course is a subjective barrier which has yet to be broken down, but from a theoretical point of view it invalidates neither the objectivity of the theory nor the theory itself.

To return now to the further development of the theory. The geometrical argument above applied only to the case where the consumer divides his expenditure between two commodities. In the numerical example above we ran ahead of ourselves, but this can be easily rectified. It need only be pointed out that a collection of things can always be treated as though they were divisible into units of a single commodity as long

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as we assume that the relative prices remain unchanged. This of course is what was done above with the baskets of food. As long as the prices of consumption goods are assumed to be given, we can bring them together into one commodity--either baskets of food, money, or purchasing power in general. This has no effect whatsoever on the classification of the effects of price into the substitution effect and the income effect. Indeed it extends the application.

Summation of the Laws of Group Demand

Our discussion so far has been devoted to the study of the demand behavior of a single individual. But this by itself is not an end. Economics is interested in the behavior of the individual only in so far as it will lead to the study of the general market. The study of individual demand then must be a means to the more inclusive study of market demand. As is to be expected, our present method enables us to make the transition without difficulty.

The properties of individual demand are almost identically those of market demand with the exception that the latter represent a cumulative effect. We have seen that the change in the individual amount demanded in response to a small change in price can be divided into the income effect and the substitution effect. The change in the group demand is the sum of changes in individual demand; it is therefore

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Our discussion so far has been devoted to the study of the demand behavior of a single individual. But this by itself is not an end. Economics is interested in the behavior of the individual only in so far as it will lead to the study of the general market. The study of individual demand then must be a means to the more inclusive study of market demand. As is to be expected, our present method enables us to make the transition without difficulty.

The properties of individual demand are almost identically those of market demand with the exception that the latter represent a cumulative effect. We have seen that the change in the individual amount demanded in response to a small change in price can be divided into the income effect and the substitution effect. The change in the group demand is the sum of changes in individual demand: it is therefore

also divisible into these two effects, the sum of individual income effects and the sum of individual substitution effects. The same propositions which evolved from the individual effects will hold for the group effects.

As the individual substitution effect works in favor of increased consumption of the commodity whose price has fallen, so also will the group substitution effect work in the same direction. Similarly as the individual income effect is not reliable, so also is the group effect not reliable; an increase in income for the group as a whole may increase or decrease the amount of a given quantity consumed depending on whether or not that commodity is inferior. It may be inferior for some members of the group and not for others. However, the total group income effect will be the sum of these negative and positive component group effects, one offsetting the other, but the stronger holding sway. Likewise it can be shown that the relative importance of these effects depends on the proportion of total group income which is spent on a particular commodity. If the group as a whole spends a large proportion of its income on the commodity then the income effect will be considerable. If the situation is reversed then it will be negligible. And finally just as it was possible for the individual income effect to be swamped by the individual substitution effect, so also is it possible for the

income effect of certain members of a group to be swamped by the substitution effect of other members. As we see, the cumulative aspect applies but the mere existence of the group instead of the individual multiplies the possibilities of offsetting and adding.

Our analysis now provides us with the ability to describe the general characteristics of the group demand curve. Ordinarily the demand curve for a commodity will slope downward, more being consumed when the price falls, except in those cases in which the commodity is an inferior good. Even when it is an inferior good, if the proportion of income spent on the commodity is small, then the income effect will be small and the curve will still slope downward. Furthermore even when the amount of income spent on an inferior good is proportionally large, the curve may still slope downward, for as we pointed out above a large negative income effect of a component part of one group may be more than offset by a large substitution effect of another component part. Since the substitution effect is always reliable and is by far the more important of the two, and since the income effect works in the same direction as the substitution effect, except in the case of inferior goods, we may conclude that in the vast majority of situations the demand curve will slope downward. This, of course, does not preclude the possibility of a positive sloping demand

curve. If the commodity is an inferior good, and the proportion of income spent on it is large, then it is quite possible for the income effect to swamp the substitution effect so that the resulting demand curve is positively sloped.[1]

The circumstances in Marshall's famous Giffen case are particularly applicable for these conditions. Here it was shown how a rise in the price of bread

> makes so large a drain on the poor laboring families and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods; and, bread being still the cheapest food which they can get and will take, they consume more and not less of it.[2]

At the low level of income here indicated the consumers had to give up some of their variety in diet and consume more bread because of the rise in the price of bread. Had the price fallen they would have consumed less bread and substituted more of other foods. This is clearly a case where the negative income effect is strong enough to more than offset the substitution effect. We can see, however, that the circumstances in this case are peculiar and not often likely to occur when one considers the whole market. Such cases are

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[1] In such a situation the marginal rate of substitution between the given commodity and other goods would be great so that there would be a tendency to substitute the commodity for other goods. A high marginal rate of substitution is characteristic of inferior goods.

[2] Alfred Marshall, <u>Principles of Economics</u> (8th edition; London: The Macmillan Company, 1922), p. 132.

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rare, but they should not be overlooked.[1]

We may conclude then that the downward sloping demand curve is predominant and that exceptions to it are rare and relatively unimportant.

This completes the theory of consumer demand in its direct development. We can now see just what has been accomplished. Starting from a given scale of preferences and given supplies of two goods we have seen how the individual will attempt to exchange these for each other, when the prices of both are given. By assuming that one of these is a composite good we have seen the relationship between one good and all others. Next we have seen the effect of price on the decisions to buy or sell and particularly in this respect we have seen the proper importance of the substitution and income effects. Finally we have aggregated these decisions by

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[1]This is particularly true if we do not regard consumers' income fixed in terms of money. If the consumer comes to market with a stock of goods, part of which he sells and part of which he consumes himself, then the income effect and substitution effect do not work the same. For the buyer only, income and substitution effects work in the same direction except in the case of inferior goods. For the seller, however, they work in opposite directions and only work in the same direction in the case of inferior goods. This is important for it is here that income effects are considerable and cannot be neglected. In the case of the buyer they can be largely neglected. This is why Marshall's constant utility of money was not such an extreme error. In the case of sellers, however, particularly since they derive large parts of their income from a particular commodity or service which they sell, income effects will be just as powerful as substitution effects and will very often be dominant.

the process of additivity so that they can be applied to groups of individuals, establishing thereby a group theory of consumer demand.

The most obvious application of this analysis is to the spending of consumer income in the satisfaction of immediate personal wants. But it is not limited to this case only. It was not intended that the goods exchanged be only consumer goods. The original limiting condition was only that the goods be objects of desire which can be arranged in an ordinal preference scale which itself is independent of price. We therefore can include in the application the supply of labor. The individual may be thought of as preferring one size income earned by one amount of labor to another size income earned by doing another amount of labor. Or the individual may prefer so much time spent in work to so much time spent in leisure. The applications both general and particular are numerous. We shall go into some of these in a later chapter.

But before leaving the theory, certain areas of exclusion must be pointed out. There are two which are important. One is the case of speculative demand. It is very often pointed out as an irregularity, how the fall in the price of a good diminishes rather than increases demand because it sets up an expectation of further fall in price. In this situation the indifference analysis does not apply, because through the expectation of fall in price, the preference scale becomes

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dependent on price.[1] (The marginal rate of substitution between the good and money ceases to be independent of price.) The other exclusion is in the field of production. Here the indifference analysis does not apply because factors of production cannot in the above manner be placed in a scale of preference. The demand for each factor is a derived demand depending on its monetary contribution to the total product. The price of the product has a direct bearing on what will be paid for each unit of the factors making up the product. The preference scale cannot be formed independent of price, so that this must be excluded. Production, however, can be treated by the use of iso-product curves which are similar geometrically to indifference curves. Treatment of this, however, is beyond the scope of our discussion.

Both of these exclusions involve the reaction of price on the scale of preference. Any problem which does not involve this reaction may be treated by the above technique.

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[1]Veblen's example of diamonds, an object of ostentatious expenditure, is a case in point. The demand for diamonds may be reduced by a fall in price because the desire depends on price and falls when price falls. The marginal rate of substitution between diamonds and money depends on price and the scale of preference therefore cannot be formed independently.

Consumer's Surplus--Restated

One of the most controversial doctrines in economic theory has been the doctrine of consumer's surplus put forth by Marshall. From his work on individual utility Marshall constructed this doctrine to throw light on certain problems of taxation. He attempted to show the existence of certain surpluses of utility which he translated into monetary terms. He could thereby show the reaction on these surpluses of specific taxes so that they could be positively or negatively evaluated. The merit of the work on taxation has not stood up because it was based on a doctrine generally considered invalid, so much so that it has been excluded almost unanimously from the main body of economic theory.[1] The development of the indifference analysis has allowed a restatement of the propositions on taxation[2] and, what is more important, it affords a means of restating the doctrine of consumer's surplus in terms which appear more generally acceptable. The theory we have been developing above is considered by its proponents useful in this respect, so that while the discussion lies off the main track of our inquiry it will be helpful to restate the doctrine of consumer's surplus in our new terms.

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[1]For criticism see H. J. Davenport, The Economics of Alfred Marshall (Ithaca, New York: Cornell University Press, 1935), pp. 101-106.

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Marshall's argument was taken from Dupuit, the original inventor of consumer's surplus.[1] Dupuit held that economics must take as a measure of utility the maximum sacrifice which each consumer would be willing to make to attain a good. Thus in Diagram 13a if OP measures price and ON quantity, then the curve Pd will be the individual demand curve showing the various quantities the individual would be willing to buy at the various prices. At quantity r the maximum sacrifice or total utility which accrues to the individual is equal to the sum of the increments, previous to r which the individual would have purchased had the price been higher. Suppose for example that rn represents 10 cents and that Or represents 20 pounds of sugar. The individual therefore purchases his 20th pound of sugar at 10 cents per pound. At price r'n' which is 8 cents, the individual will purchase 30 pounds of sugar. But for the 20th pound he would have paid 10 cents so that at 8 cents there is a consumer's surplus of 2 cents. The total utility at 20 pounds is equal to the sum of what he would have paid for the increments previous to the 20th pound, rather than go without them, or the area OPnr. The cost at 20 pounds is indicated by what he actually pays OPnr, leaving a surplus

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[1]Jules Dupuit was a French engineer interested in the economic aspects of engineering. His work appeared originally in the Annales des Points et Chaussées in 1844 and was very inaccessible until translated by Mario de Bernardi under its original title, "De l'utilité et de sa mesure," in Paris, 1934.

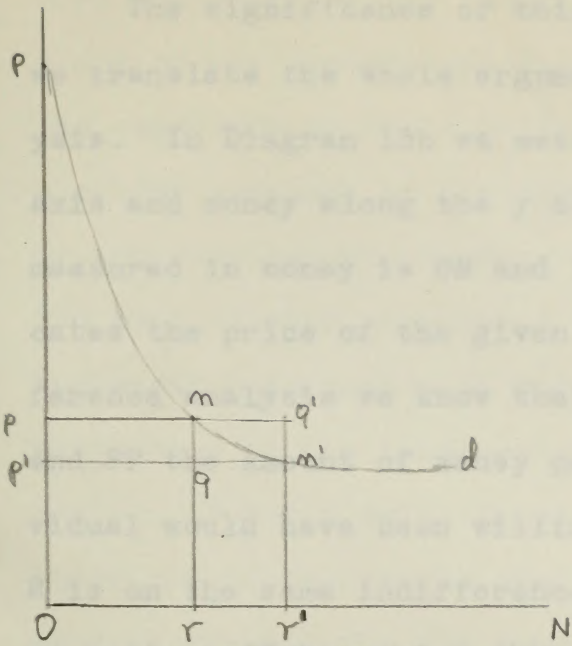

DIAGRAM 13a

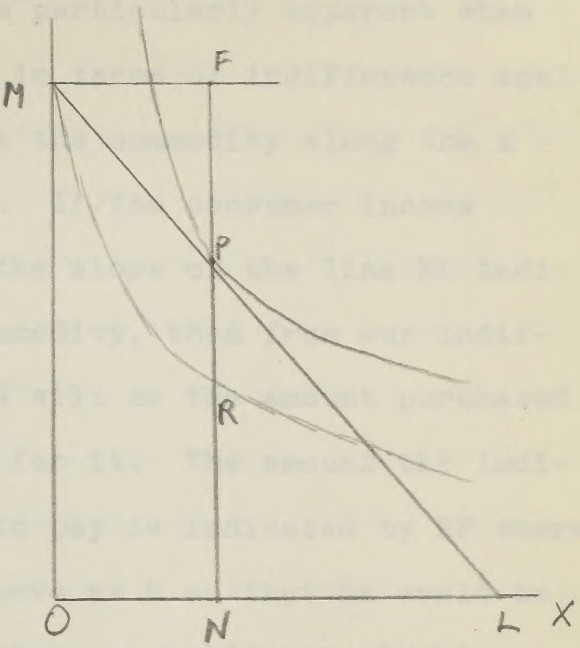

DIAGRAM 13b

equal to pPn. Similarly at r' the consumer's surplus would be represented by p'Pn'. This is the theory explained by Dupuit[1] and also by Marshall and through which both arrived at the same results, but there is a qualification to be made which Marshall was very particular to include, namely that the marginal utility of money throughout must be assumed constant. This is a basic assumption on which is based the construction of the demand curve and without which different increments of utility would have a confused meaning. Marshall was very precise in his development and he therefore included this assumption.[2]

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[1]Ibid., pp. 62-65 for discussion and diagrams 1, 2, 3, in appendix.

[2]Marshall, Principles of Economics, op. cit., pp. 126, 128, 132, also Appendix, Note VI, p. 842.

The significance of this is particularly apparent when we translate the whole argument in terms of indifference analysis. In Diagram 13b we measure the commodity along the x axis and money along the y axis. If the consumer income measured in money is OM and if the slope of the line ML indicates the price of the given commodity, then from our indifference analysis we know that ON will be the amount purchased and PF the amount of money paid for it. The amount the individual would have been willing to pay is indicated by RF where R is on the same indifference curve as M so that he would be no better off by making the purchase or holding on to his money. The consumer would have been willing to pay RF. He actually pays PF. Consumer's surplus is therefore the excess of one over the other or RP. The indifference diagram thus affords a means of expressing in monetary terms the gain accruing to the consumer over what he would have been willing to pay, without resort to the utility concept and independent of any assumption about the marginal utility of money.

But the marginal utility of money is important. Let us examine the implications. We have by two different means arrived at a monetary expression of the same surplus. It must follow then that RP equals the area pPn under the individual demand curve; but is this always so? Only when the marginal utility of money is assumed constant, that is, only

when the slope of the indifference curve at R is equal to the slope of the indifference curve at P. This is the corresponding condition on the indifference scale. From it we see that a slight movement to the right of P will increase PF and RF by the same amounts leaving consumer's surplus RP the same. And this is what happens on the demand curve. At point n the increased increment will increase the total amount spent, corresponding to the increment of FP, by an amount equal to rng'r'. But the total utility corresponding to RF is also built out of increments similar to rnq'r', so that consumer's surplus pPn remains the same.

This becomes apparent when we make the transition to group demand. Let us assume that Pd in Diagram 13a represents the amounts that different individuals would be willing to pay at different prices. Then similarly the maximum sacrifice or total utility derived by all those who purchase at quantity r is equal to the sum of the increments previous to r which some would have purchased had the price been higher. The individual who buys the r^{th} pound pays rn for it. His total utility or maximum sacrifice is the thin line rn. For the group the total utility is indicated by the sum of increments similar to rn previous to r. The consumer's surplus then is the excess of total utility OPnr over cost OPnr or pPn. Now comparing this with the indifference diagram we find that the area OPnr is made up of the sum of increments similar to

when the slope of the indifference curve at R is equal to the slope of the indifference curve at P. This is the corresponding condition on the indifference scale. From it we see that a slight movement to the right of P will increase PF and RF by the same amounts leaving consumer's surplus RF the same. and this is what happens on the demand curve. At point m the increased increment will increase the total amount spent, corresponding to the increment of PF, by an amount equal to mrq'r'. But the total utility corresponding to RF is also built out of increments similar to mrq'r', so that consumer's surplus pPm remains the same.

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RF at various slopes of ML (various prices); Opnr is made up of the sum of individual costs or the sum of increments of PF. If the slope at P is always equal to the slope at R then the corresponding areas of the demand curve will always equal the sum of the corresponding increments on the indifference diagram. If the slope is different then these areas will not be the same and another effect will enter--this indicated as the income effect on the indifference diagram. This is the precise meaning of constant marginal utility of money. We saw it previously and we see it here again. It enabled Marshall to neglect the income effect. It is true that this difference is likely to be small, the less important the commodity considered is in the consumer's budget; but even if the proportion of income spent is small, it can still be important if RP itself is large so that the loss of opportunity to buy will be equivalent to a large loss of income. The indifference analysis thus eliminates that weakness in Marshall's argument since it expresses the gain monetarily without any assumption concerning marginal utility of money.

Under the Marshallian analysis consumer's surplus was a means of expressing in terms of money income the result of a fall in price. But it may now be viewed differently--as the compensating variation in income, the loss of which would just offset the fall in price, and leave the consumer no better off than before. This is equivalent to the former with the

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exception that it includes the income effect. And it can be shown that the corresponding variation in income cannot be less than a minimum amount and is ordinarily greater than that amount.

If for example oranges are 2 cents each and at that price 6 oranges are bought, what will be the compensating variation in income when the price is 1 cent and 10 are bought? We know that it cannot be less than 6 cents for if the individual bought 6 oranges at 1 cent and we assumed his income had been reduced by 6 cents then he would still be as well off as before. What had been his previous position is still open to him. In all probability he will substitute some quantity of oranges for some other things and make himself better off. But if he can lose 6 cents and remain as well off as before, then 6 cents is less than the compensating variation. He would have to lose more than 6 cents in order to be just as well off as before, but just how much more cannot be indicated since the information about the indifference curves in the new situation is not available. All that can be definitely known is the minimum variation, and fortunately for most of the propositions in which the concept is used, this is all that is necessary.

However, it is theoretically possible to make the assumption that the individual's tastes remain unchanged even after a rise or fall in price. By doing this it becomes possible

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However, it is theoretically possible to make the assumption that the individual's tastes remain unchanged even after a rise or fall in price. By doing this it becomes possible

to compare two equilibrium situations and to indicate graphically the compensating variation, but it must be remembered that whatever measures are indicated fall within the limitation of this assumption.

In Diagram 14 we see an individual's indifference curves between commodity X and Y, money. It is assumed that all prices other than X are constant so that Y also represents the quantity of all other goods. Individual income is taken as constant, OM. The two equilibrium situations will appear at points A and B, points of maximum satisfaction resulting from the price changes from MP_1 to MP_2 or vice versa. What we want to indicate is the gain to the buyer when the price falls from P_1 to P_2 and the loss when the price rises from P_1 to P_2 and the buyer moves back from B to A.

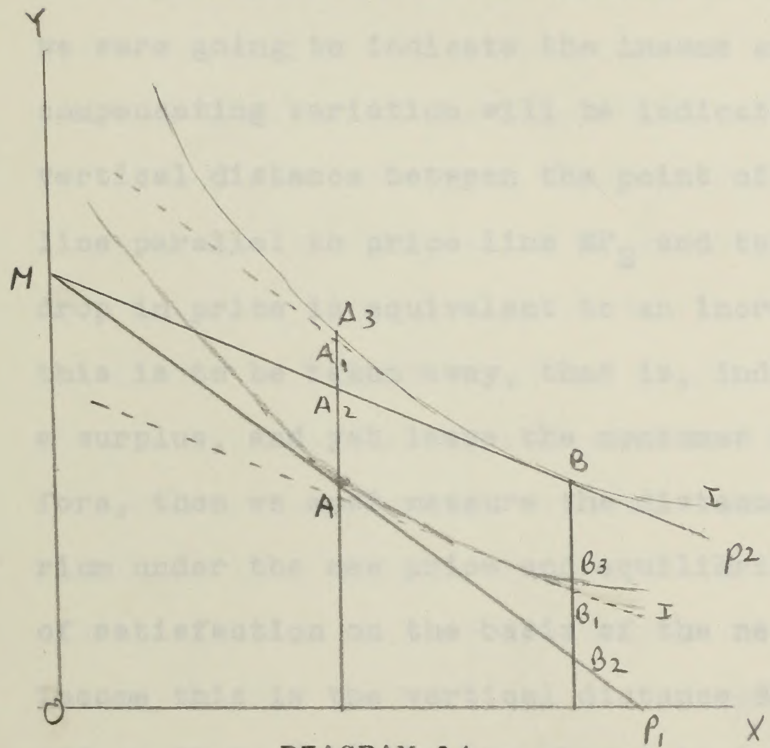

DIAGRAM 14

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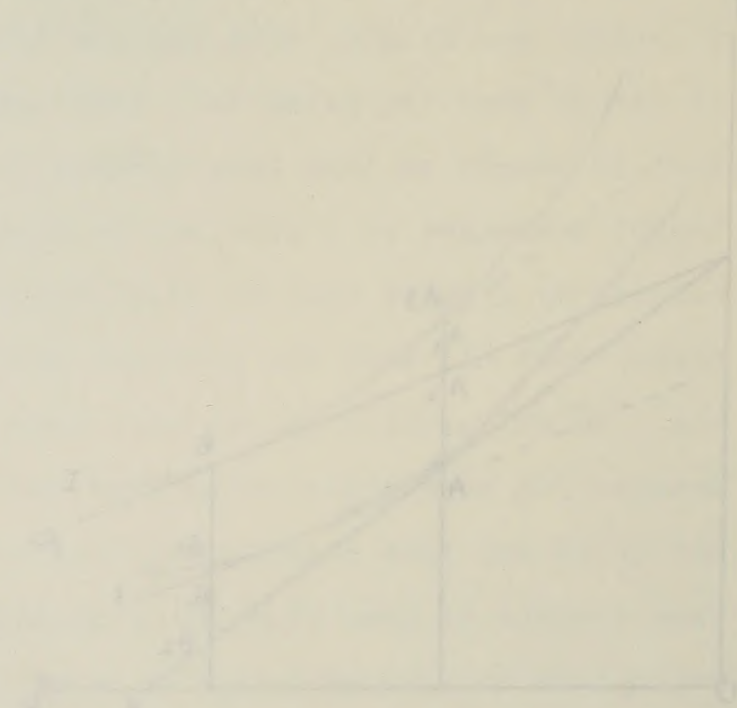

DIAGRAM 14

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at the new price just as attractive and no more attractive than trading at the old price with original income. Or in later Hicksian terminology it is the "price compensating variation" resulting from a fall or rise in price.[1] If in our diagram we draw in the parallels to the price line as if we were going to indicate the income effect, then the price compensating variation will be indicated by BB_1. BB_1 is the vertical distance between the point of equilibrium B and the line parallel to price line MP_2 and tangent to curve I. The drop in price is equivalent to an increase in income, but if this is to be taken away, that is, indicated in the form of a surplus, and yet leave the consumer no better off than before, then we must measure the distance between the equilibrium under the new price and equilibrium at the previous level of satisfaction on the basis of the new price. In terms of income this is the vertical distance BB_1 which may be marked off on OM to indicate the change in income. This is not a minimum variation as explained above but rather an indication of the exact variation. It is possible to indicate it thus only because the assumption was made at the start.

In Value and Capital and in a subsequent article defending consumer's surplus[2] Hicks felt that the change in income

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[1] J. R. Hicks, "The Four Consumer Surpluses," Review of Economic Studies, 11:31-41, Winter, 1943.

[2] J. R. Hicks, "The Rehabilitation of Consumer's Surplus," Review of Economic Studies, 8:108-116, February, 1941.

indicated by the price compensating variation was what Marshall meant by a change in consumer's surplus. (It will be remembered that Marshall indicated consumer's surplus by the triangle under the demand curve.) But Hicks was criticized for this by Professor Henderson who pointed out that under the above assumption four expressions of consumer's surplus were possible instead of one which Hicks indicated, but that only one of the four resembled Marshall's measure.[1] This is indicated by the line BB_3 which Hicks in his 1943 article calls the "quantity compensating variation."[2] This is defined as the amount of income which the consumer would have to lose or gain after adjusting his purchases of X to the new price in order to be no better off than before. Hicks agreed with Henderson that this was the better measure and the one which was the closest to what Marshall meant by consumer's surplus. However in his 1943 article he reverts back to the area between the demand curve and the price axis within the range of the price change, as Marshall's measure. In his private correspondence Hicks finally clears this up.[3] He states,

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[1]A. Henderson, "Consumer's Surplus and the Compensating Variation," Review of Economic Studies, 8:117-121, February, 1941. Henderson pointed out the measures. Hicks gave them the names by which they now go.

[2]Hicks, "The Four Consumer Surpluses," op. cit.

[3]See James N. Morgan, "Measurement of Gains and Losses," Quarterly Journal of Economics, February, 1948, pp. 290-291. Hicks' private letter to Morgan is reprinted in this article.

> I think I would now agree with Henderson that the quantity variation is the strict meaning of Marshall's concept of consumer's surplus. But when I talked about the Marshall measure I meant the approximation which Marshall gave to this concept, namely the triangle under the demand curve. The price variation is, I now think, quite foreign to Marshall's theory, but it is a concept which one naturally drops into when trying to generalize and it is important as providing a link between consumer's surplus and index number theory.

So far we have considered only two of the measures pointed out by Henderson. The other two are the quantity equivalent variation and the price equivalent variation. The quantity equivalent variation measures the gain or loss in income on the original level of consumption which is equivalent (in utility) to the fall or rise in price. This is equal to the vertical distance between the indifference curves at the original level of consumption, AA_3 for a fall in price and BB_3 for a rise in price. It is to be noted that these measures are identical with the quantity compensating variation except that they are reversed for a price rise and price fall. The price equivalent variation measures the gain or loss in income which is equivalent to the fall or rise in the price. This is measured by AA_1 for a fall in price and BB_1 for a rise in price. Here also it is to be noted that the measure is the same as the price compensating variation except that it is reversed for a fall or rise in price. The reason for the reversal lies in the fact that we are comparing two different measures, price and quantity, from two different

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points of view. Four measures result which are somewhat alike, but are different since four distinct things are being measured. If additional limiting assumptions were to be made other measures would be possible.

The doctrine of consumer's surplus as developed above is a translation of the concept developed by Marshall onto the indifference map. Herein lies its interest for us. But just as the work of Marshall was questioned, so also has the translation onto the indifference map been questioned. And to a large extent the criticism is justified. The indifference map assumes only a series of ordinal preference curves and therefore little importance is to be attached to a numerical measure of the gain or loss from a price change. Within the limits of the above assumption one can compare as above the gain between two price situations, but it is impossible to compare with any advantage, the gain between two price changes with the gain between two other price changes.[1] And moreover, those propositions which are explained in terms of consumer's surplus, particularly those relating to the burden of taxation can be stated independently of any numerical measure of gain. Most of them can be stated purely in terms of indifference analysis. As was the case under Marshall's theory there

[1]See P. A. Samuelson, Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947), p. 198, especially footnote 35.

points of view. Four measures result which are somewhat alike, but are different since four distinct things are being measured. If additional limiting assumptions were to be made other measures would be possible.

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appears little need for the concept even aside from its validity. It appears then that the concept is in about the same state as it was before Hicks revived it. Although he did restate it, no fruitful results have followed.

CHAPTER IV

RELATED GOODS

Early Historical Development

In the discussion to this point I have been very careful to avoid the problem of related goods. This has been done primarily for the sake of clarity, because, for the most part, this problem has been closely interwoven with the development of indifference analysis and there has resulted a confusion which has obscured the real significance of each. One of the main purposes in treating the problem separately is to place it in its proper relationship to the whole theory of demand and thereby do away with much of the prevailing confusion. As we retrace the development this will become apparent.

Previous to the development of indifference analysis very little was said about related goods. It was recognized that the demand for one good is not dependent on that good alone, but is more often tied to the demand for other goods. Particularly in the productive process it was recognized that goods are jointly related. This arose from the determination of joint costs of production. But aside from recognizing the joint relationship little attention was given the problem and no exact theory was worked out. John Stuart Mill, for

example, met the problem in his discussion of joint costs of production.[1] He recognized the effect of joint cost on demand, but to him the problem was of the extraordinary variety. He includes it in a chapter entitled "Some Peculiar Cases of Value." Marshall also discussed joint demand as affected by the productive process. For him the demand of each of several complementary things is derived from the service which they jointly render, in the production of some ultimate product.[2] The joint relationship exists, but it is set aside in order to isolate each factor for separate study.[3] The problem was also discussed by certain members of the utility school incidental to their main theme. Jevons, for example, treats it, but he reprimands Mill for considering it a "peculiar case." On the contrary most commodities are produced jointly with minor commodities.[4] Friedrich Wieser also considers it. In treating the value of factors of production he uses the principle of complementary goods to show "the elements that are bound up" in production.[5] All these early attempts merely

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[1]John Stuart Mill, Principles of Political Economy (New York: Appleton and Company, 1880), Book III, Ch. XVI.

[2]Marshall, Principles of Economics, op. cit., p. 381.

[3]Ibid., footnote 2, p. 383.

[4]W. S. Jevons, Theory of Political Economy (4th edition; London: The Macmillan Company, 1924), p. 198.

[5]Friedrich Von Wieser, Natural Value, p. 87.

recognized a complementary relationship involved in production. That the complementary relationship was connected to the substitution relationship or that these both carried over to demand was not yet seen. Thus no concise theory was attempted.

Among the very first to go beyond the mere productive relationship were Auspitz and Lieben. They recognized substitution as it was related to complementarity and more important they treated the relationships on the demand side. Their discussion includes a section on complementary and competitive articles and another on the effects of changes in price of such articles. The two relationships, they say, are in a sense opposed. An increase in the consumption of article A generally increases the consumption of those articles which serve to complete the enjoyment desired by the consumption of A. On the other hand for those articles which are competitive with A the consumption will decrease if that of A increases.[1] Here are perhaps the first precise definitions of complementary and competitive goods. Complementarity according to this definition infers that goods are so tied up that one good must have some other for the two to produce a given enjoyment. The two goods together complete the enjoyment, such as sugar and coffee. Competitive goods on the other hand are rival goods;

[1]Auspitz und Lieben, Untersuchungen über die Theorie der Preises, op. cit., section 36, p. 154.

they compete with each other for the same enjoyment such as wine, beer, or whiskey. Thus an increase in the consumption of coffee will always result in an increase in the quantity of sugar complementary with it; but if the same individual instead of increasing his consumption of coffee decreases it and instead increases his consumption of tea, it is possible (depending on the relative price of these goods) for the total consumption of sugar to decrease.[1] Similarly an increase in the individual's consumption of beer will result in a decrease of the consumption of wine and whiskey, but it is possible for the decrease in whiskey to be so great that there actually will be an increase in the consumption of wine.

We thus see here a complete understanding by Messrs. Auspitz and Lieben of the different complex relationships which are possible. By no means does their treatment go into every possible relationship, but it does include a great many and there is a remarkable similarity between their results and the more thorough results of Hicks which we shall come to below. They point out how a drop in the price of an article substitutable for A will cause a decrease in the immediate consumption of A while a drop in the price of an article complementary with A may result in an increase in the consumption of A. The extent of the effect of the fall

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[1]*Ibid.*, p. 155.

in price will depend on how much of A one is using.[1] Furthermore they show the effects of changes in consumption on total utility. For complementary articles important in total consumption, a decrease or increase in the price of one will lower or raise total consumption as the case may be. In the case of a competitive article a decrease in price will increase the utility of the article itself and make it more advantageous relative to others. But regardless of the relationship, the increase in total utility will depend on the relative importance in consumption of the article whose price has fallen. But as we explained earlier, as ingenious as these men were, they still clung to the utility idea and they thus did not develop the insight necessary for the next step in the development. They recognized the many possible relationships. While they did not discuss price effects on the basis of a limited income, they did see that the importance of an article in the individual's consumption had a great bearing on the effects of price on related goods. Developments in any theory do not take place all at once. One cannot expect the final result immediately. Thus we must recognize the foundation laid by these men in this problem and continue further to trace the development of the concepts set out by them.

The insight necessary for further advancement in the

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[1] *Ibid.*, Section 41, pp. 171-172.

theory came about as a result of the development of the indifference device. This tool was not constructed specifically for the problem of related goods. In economics it is generally impossible to construct a tool for a particular problem, but it was seen almost immediately that the tool lent itself to the treatment of related goods and it has become to a very great extent associated with that problem. We ask naturally therefore what there is in the indifference analysis which made it so adaptable to and focused so much attention on that particular problem. The reason for this lies in the very nature of the indifference curve itself. It will be remembered that in Marshall's work and also in the work of the early utility proponents, utility was considered as the function of one good only. It had been recognized that the utility of a particular good does not exist independent of itself, but there was no means geometric or mathematic to treat the relationship of the utility of one good to that of another. The development of indifference curves supplied this means at once geometric and mathematic; for it treated goods one in combination with another and thereby allowed the utility of one to be considered as it affected and was affected by the utility of another. This immediately directed attention to the problem of related goods as it supplied the device to explain those consumption relationships which had been previously recognized but inadequately explained.

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Related Goods Under the Indifference Analysis

All studies in economics are directed toward the explanation of phenomena as they occur in everyday life. Unfortunately the economist is limited in his use of the inductive method. Society is his laboratory and it is therefore almost impossible to achieve results from strictly controlled conditions. He has therefore been forced to fall back on the deductive method of reasoning--to start with given hypotheses, reason logically, and then check his results with what actually takes place and make the necessary correction in his original hypotheses. It very often occurs under this method that the theory itself is thoroughly correct in so far as its logical reasoning is concerned, but that the basic assumptions do not correspond to what actually takes place or that other forces are operating which have not been accounted for in the original assumptions. This has been very true with the problem of related goods. The development has been achieved for the most part by the use of the deductive method. Whole theories have been worked out which in themsevles are flawless. These have been verified by the use of mathematics, a tool which has been a very considerable aid in deductive reasoning. But these theories have proved to be inadequate; not because they are in themselves wrong, but because their assumptions either have not corresponded to actual conditions or have excluded factors which have a great bearing on the theory. This is

the result of the hybrid method (the combination of the inductive and deductive methods) which economics under the present state of knowledge has been forced to use. Aware then of the limitations in the methods used, we must examine with care the definitions which serve as hypotheses for the different theories that we may understand how they differ and to what extent one progresses over the other.

The definitions of Auspitz and Lieben were used for a theory which did not have the benefit of indifference analysis. In a theory developed by Francis Edgeworth one would naturally look for definitions of significance since he was the first to establish the indifference curve. Such definitions are given, but their significance is not what is expected. It will be remembered that Edgeworth's treatment considered the utilities of two goods, one of which is given and the other, acquired for the former, the acquisition yielding utility and the sacrifice disutility.[1] The indifference curve in this form did not consider the positive utilities of two goods in conjunction with each other. Edgeworth did not see how readily the indifference curve could be applied to this problem and his definitions were therefore not part of an integrated theory of related goods. The definitions he gives appeared eight years after his <u>Mathematical Psychics</u> in an article on the pure theory of

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[1]See above pages 10, 43.

monopoly.[1]

They are given in pure mathematic terms which when translated take on the following meaning: Y is complementary with X when an increase in the supply of X raises the marginal utility of Y. Y is a substitute for X when an increase in the supply of X lowers the marginal utility of Y.[2] Goods are complementary when their marginal utilities move in the same direction and substitutable when they move in opposite directions. Movements in either will have the same effect on each other or,in other words, the relationships are completely reversible. If Y is complementary with X then X is complementary with Y. The factors considered in these definitions are the quantity of each good and its marginal utility. But the quantity demanded is a function of the price so that price is also a factor; indeed Edgeworth held that variations in utility can be measured monetarily.[3] He has included then these three

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[1]Edgeworth, Papers Relating to Political Economy, The Pure Theory of Monopoly, op. cit., Vol. II, p. 117. Edgeworth's use of complementary and rival goods is in the proof of the proposition that when two or more monopolists are dealing with competitive groups economic equilibrium is indeterminate. This use is entirely irrelevant to the present discussion and therefore will not be treated here.

[2]Mathematically if $\frac{d^2\ Fr}{d\ x\ dy}$ is negative the goods are substitutes. When $\frac{d^2\ Fr}{d\ x\ dy}$ is positive the goods are complementary. For the translation see Hicks, Value and Capital, p. 42.

[3]Papers Relating to Political Economy, op. cit., p. 117. Edgeworth refers to Dupuit as authority for this statement.

factors in Marshallian fashion and has used them for a particular purpose, but from our point of view it is impossible to evaluate them. His use in the theory of monopoly is very vaguely related to this discussion and furthermore the definitions were not used in an Edgeworth theory of related goods. It is to be noted however that with slight change they are the same as the definitions set out by Pareto. Indeed by some they are associated together as the Edgeworth-Pareto definitions. We shall therefore have occasion to meet them again when we shall evaluate them in the light of a system of reasoning built on their foundation.

Fisher

The first complete theory of related goods was worked out by Irving Fisher. It will be remembered from our previous discussion that Fisher's development of the indifference curve while appearing at the same time as Edgeworth's, was independent of it and entirely original. It therefore turned in a slightly different direction. Fisher actually in his development produced the curve as we use it today, based of course on the utility concept. He first considered the utility of a given commodity as a function of that commodity alone (Part I, Mathematical Investigations), but when he came to the indifference analysis (Part II), he changed his assumption and assumed that the change in the marginal utility of a given quantity of a commodity changes the marginal utility of a

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given amount of another commodity.[1] From the beginning then his work was a step advanced over that of Edgeworth, for he considered marginal utilities positively as they affect each other.

This change in the basic assumption immediately raised questions about the various relationships possible between the marginal utilities of different goods; that is, it posed the problem of related goods. Therefore simultaneous with the establishment of indifference curves it became necessary to establish definitions to cover these relationships. Since the curves themselves are derived from surfaces indicating changing marginal utilities, there must be some property in them which also indicates the same thing. Thus Fisher gives the following definition. "The essential quality of substitute or competing articles is that the marginal utilities or the prices of the quantities actually produced and consumed tend to maintain a constant ratio."[2] Perfect substitutes are defined such that this ratio is constant. "The essential attribute of completing articles is that the ratio of the _quantities_ actually produced and consumed tends to be constant."

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[1]Fisher, _Mathematical Investigations in the Theory of Value and Prices_, _op. cit._, p. 64.

[2]_Ibid._, p. 65. Previous to the development of indifference curves members of the utility school (Jevons, Walras) and also Marshall had shown that marginal utility was proportional to price. Thus the equivalence in the quotation.

And perfectly completing articles are defined such that this ratio also is constant, "as many shoe strings as shoes for instance, irrespective of cost."[1] There is a distinct difference between these definitions to be carefully noted. One is based on the ratio of marginal utilities and the other on the ratio of quantities. Evidently Fisher felt that the essential attribute of complementarity lay in the proportional numbers of each commodity consumed and not in the change in the ratio of marginal utilities. The very nature of complementarity is in association and he therefore shifted to quantities rather than marginal utilities.

It will be remembered from our previous discussion that Fisher's total utility surface from which the indifference curves were directly established, was derived from two derivative surfaces, one indicating the marginal utility of A, B constant, the other indicating the marginal utility of B, A constant.[2] The primitive surface, usually in the shape of a hill, contained the relationships of the previous two. Now if these are carried over in the indifference curves then what is the property of the curves which indicates the relationships or, in other words, to what extent and just how do they indicate substitution and complementarity? The answer

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[1] <u>Ibid</u>., p. 66.

[2] See above pages 20-21.

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is found in the original definitions and in the logic of the curves themselves.

Perfectly competing or substitutable goods are defined such that the ratio of their marginal utilities is constant. Now we know from previous discussion that the slope of the line at any point on an indifference curve indicates the marginal utility of one good, the other being held constant. For the ratio of the marginal utilities of the two goods to be constant at every point on the curve, the curve must reduce to a straight line and the system to a series of parallel straight lines. "For perfect substitutes the curves reduce to parallel straight lines whose intercepts on the A and B axes are inversely proportional to the fixed ratio of their marginal utilities."[1] As an example of this Fisher gives the case of "Lehigh" and "Lakawana" anthracite coal which are nearly perfect substitutes. "If it cost nothing the individual would indifferently consume the quantity of one or the other or any combination of the two on the straight line 99 inclined in this case at 45 degrees."[2] Here the goods are perfectly substitutable in such a way that as one moves along a given curve an equal number of units of one will exchange for an equal number of units of the other. But at this point

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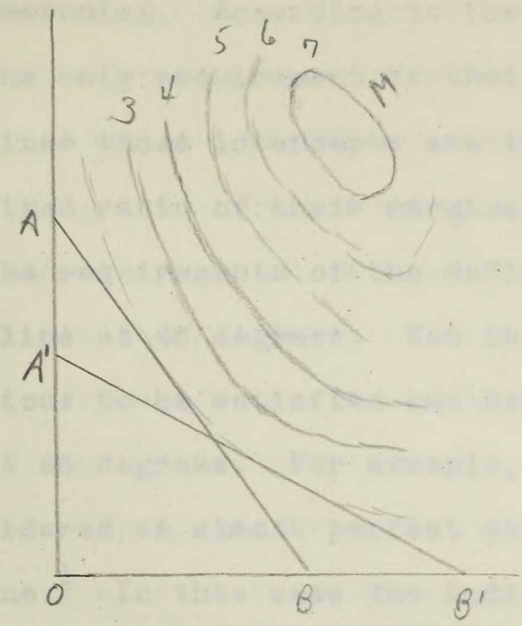
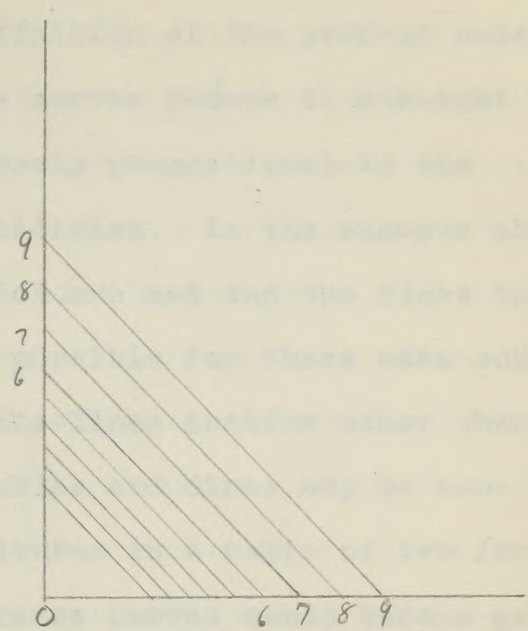
[1]Fisher, Mathematical Investigations in the Theory of Value and Prices, op. cit., p. 71.

[2]Ibid.



COMPLETING GOODS



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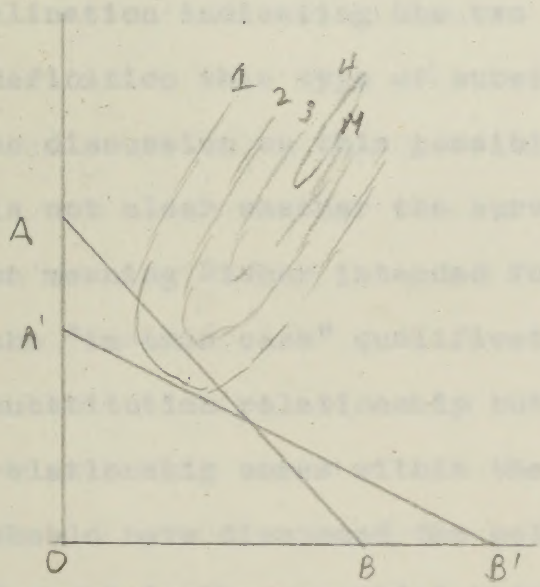

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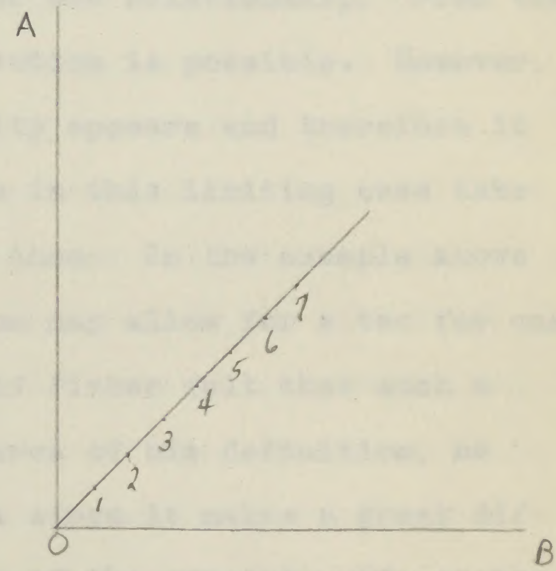

PERFECTLY COMPLETING GOODS

DIAGRAM 15

there is a certain ambiguity which clouds the logic of the reasoning. According to the definition of the perfect case the only requirement is that the curves reduce to straight lines whose intercepts are inversely proportional to the fixed ratio of their marginal utilities. In the example above the requirements of the definition are met and the lines incline at 45 degrees. Yet it is possible for these same conditions to be satisfied and have the lines incline other than at 45 degrees. For example, nickles and dimes may be considered as almost perfect substitutes in a ratio of two for one.[1] In this case the indifference curves would become parallel lines inclining at roughly 34 degrees, the degree of inclination indicating the two for one relationship. From the definition this type of substitution is possible. However, no discussion on this possibility appears and therefore it is not clear whether the curves in this limiting case take on meaning Fisher intended for them. In the example above the "in this case" qualification may allow for a two for one substitution relationship but if Fisher felt that such a relationship comes within the area of his definition, he should have discussed the point since it makes a great difference in the essential nature of the concept. If, on the

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other hand, he felt that perfect substitution can occur only on a one for one basis, then his definition should state that the ratios of the marginal utilities tend to maintain a constant ratio of unity. It may be that Fisher himself was not quite sure about the precise meaning of the concept. At any rate it is not clear from his discussion just what was intended. In spite of this, however, it should be recognized that this was the first attempt made in this direction and as such it pointed the way. From it we can derive a basic understanding of the concept and how it is to be treated with the use of the indifference tool. We see clearly that the essential characteristic of substitution is indicated by the flatness or lack of flatness of the indifference curves. The conclusion must follow that for articles which are more or less substitutable the curves will be more or less flat.

For completing articles the reasoning is similar. The more curvature to the curves the more will the articles be complementary. This follows somewhat from his definition of the limiting case. The essential nature of perfect complementarity is such that the ratio of the quantities consumed is constant. Here the "whole family of indifference curves" reduces to a straight line passing through the origin. If we regard right shoes and left shoes as two distinct commodities, then the desire for right shoes vanishes as long as left shoes are not admitted. The marginal utility for

right shoes has no application though that for pairs of shoes has.[1] In the diagram which Fisher uses to illustrate this case his line of the family of curves is inclined at a 45 degree angle. As in the case of substitution the discussion here is inadequate, but for clarity it must be stated that it is quite possible for goods to be associated in any quantitative relationship. For example, some people use two teaspoons of sugar for each cup of coffee. Assuming that this were a perfect case the line of the curves would be inclined at roughly 34 degrees rather than 45. Here again the discussion is not clear.

It is quite evident that while Fisher felt he had to treat the problem, he did so only in a cursory fashion. It is not that his reasoning does not follow from his definitions but that it is very incomplete; also even aside from the ambiguity which we have just treated, there are many other possibilities which are not accounted for. Complete relationships between the limiting cases are not explained. For example, what happens to marginal utilities in the completing case is not answered. The relationship of degrees of substitutability and complementarity is vague. What Fisher has done is to present a tool which has possibilities, but he admits that

[1]Fisher, Mathematical Investigations in the Theory of Value and Prices, op. cit., p. 71.

the relationships are not simple. "Articles may be competing at some combinations and completing at others."[1] He suggests that statistical inquiries might be made to clear up some of the difficulties. It is apparent that he himself was not too clear about the problem and that there are many intricacies still to be worked out.

Yet in spite of this there is a definite value to Fisher's work. He has set up definitions and then reasoned on the basis of them. While his reasoning is not as complete as one would like, to the extent that he has carried it out, it is correct. Whether the results or the definitions on which these results are based are adequate is quite another question. Obviously there is room for improvement in the establishment of definitions which more adequately describe the true relationships. But Fisher's work is a step forward. It contains the seeds to the theory which was to flourish later.

A further point is necessary before we leave the discussion of Fisher. It will be recalled from our previous inquiry that Fisher sought to objectivize the theory of value, and that he thought he had attained this by the use of the indifference analysis. Total utility was done away with. The economic world was to be filled only with lines of force or "maximum directions" in the orbit of which all economic

[1] Ibid., p. 76.

motivation transpired.[1] Indifference analysis, then, became the tool by which objective theory could prevail. To a large extent this was true, for it enabled Fisher to do away with many of the purely hedonistic elements which had been firmly embedded in past theory; but it was not entirely so. There was still something lacking in his objectivity. Particularly is this apparent in his treatment of related goods. The definition of substitutability is framed in terms of marginal utility, which is certainly a utility concept. The definition of complementarity is based on the ratio of quantity. But while this itself is not utilitarian, the area between the limiting cases is in some manner dependent on marginal utility. The line of demarcation where marginal utility ceases to be factor and quantity begins to dominate is not thoroughly explained, but there is somehow a tie-up, and in it the utility concept definitely seems to predominate. The significance of the objectivity is therefore diminished, for while some things can be explained by lines of force, the theory of related goods had to be explained in utilitarian terms. To this extent objectivity is lacking. If it is possible to work out an objective theory of value, then certainly the problem of related goods, which is only one small phase of this theory, should be able to be stated in the same objective terms. From the

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[1] See above, pages 23-24.

nature of Fisher's assumptions this was impossible and therefore a truly objective use of the indifference tool could not be accomplished.

Pareto

Economists have come to associate the indifference tool with the problem of related goods, and with these they have also associated the name of Pareto, for it was he who developed the first significant theory of related goods with the aid of indifference analysis. It almost appears that Pareto developed the indifference curve expressly for its application in this problem. We know, of course, that this was not so. Pareto took the tool from Edgeworth and after significant alteration he saw in it a valuable tool for the explanation of related goods. His accomplishment in this respect consists in the fact that he saw immediately that it was a means of treating the utility of one good in relation to that of another and that he proceeded to work out precise relationships. To a considerable extent this was also true of Fisher, but Pareto attached much more importance to the aspect of dependence and his treatment is thus much further developed. His discussion accounts for many of the intricacies which Fisher felt might be cleared up by statistical inquiry. His conceptual discussion of complementarity and substitutability are classic. His use of the indifference curves and particularly the

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thoroughness of his theory account for the long period of time in which it went unchallenged.

It will be remembered from our previous discussion that the theory of economic equilibrium was established by resolving equilibrium price out of the conflict of tastes and obstacles. The same over-all results had been attained through a different channel by Marshall, but Pareto, through the use of indifference analysis, achieved in many respects a more inclusive theory. Particularly is this so with the theory of individual consumer demand where, besides attaining fuller results, he was able to treat the problem of related goods which Marshall mentioned only incidentally as a relevant phase of individual demand.

As we did in the discussion on Fisher, we begin first by examining basic definitions. But in the case of Pareto, there are certain basic conceptual considerations to be examined which account for his definitions. It is there to which we turn.[1]

The utility (ophélimité), which is attained from the consumption of a given quantity of goods, may be dependent or independent of other goods consumed. In the case of independent goods the utility will be the same regardless of what other goods are consumed. This was the situation which

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[1]The following discussion of the conceptual nature of the complementary and substitute relationships is based on Chapter IV of the Manuel d'Economie Politique, Sections 1-42.

Marshall established by definition and from which he evolved the law of diminishing marginal utility. But this situation does not always prevail. Usually the relationship is one of dependence; that is, the utility attained by a given consumption depends on the consumption of other goods. Pareto distinguishes two types of dependence: that which arises from the fact that the utility of one good is in direct relation with that of another, and that which arises from the fact that one good can be substituted for another and produce the same individual sensation. Broadly speaking we call the first complementarity and the second substitution.

Complementarity is a relationship of necessity; that is, for goods to produce utility they must be joined to others in the consumption process. The extent of the dependence will depend on the accompanying circumstances. A person dying of hunger will not demand fine service wear if he is given food. He probably will ask for food regardless of the service wear or any other trimmings which usually accompany food. Here the dependence is destroyed because of an accompanying circumstance. But similarly it is possible for peculiar dependencies to be established because of extenuating circumstances. In general while peculiar conditions are recognized as having an effect on the types of dependence, if the effect is not unusual, if the variations are small, these conditions can be disregarded and the relationship simplified so that utility

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of one good depends only on the fact that it must be associated with an other in a certain quantitative measure.

But even on this basis limitation is necessary. The whole concept of complementary goods can be a very extended one. For example, sugar and coffee may be considered dependent on each other in a quantitative relationship. But so is the cup, saucer, table, chair, house, and land, which are also used so that coffee or sugar may be consumed. Here we must limit the concept, because the extended cases while plausible have little effect. In reality, the utility of houses has little effect on the utility of sugar, and a change in the price of one will have no bearing on the price of the other. What we do is to recognize that the extended relationship is present, but when the effects are so minimized, when the variations are so small, as to be entirely insignificant, we merely treat the other goods as given and consider only the quantitative aspects.

Substitution is also a relationship of dependence but in a different manner. Man can clothe himself with cotton, wool, rayon, and silk. To a certain extent these are all substitutes and there is a general preference of one for the other. But the preference is a particular type. It refers not to taste alone, but also to the particular need to be satisfied. Cotton and silk, for example, can be used interchangeably in certain circumstances. Generally silk is preferred to cotton, but for many needs cotton will be superior to silk; that is,

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it will do a better job than silk. A given consumption combination will then be effected not only by taste, but also by the need of the consumer. If we consider just tastes alone, then the situation is given on the indifference curve and a theory of substitutable goods is not necessary. But the fact that people consider both effects the shape of indifference curves and thus necessitates a separate treatment for related goods.

Also out of this is born a hierarchy of goods. If A B C are capable of satisfying a particular need, the amounts of each used will depend on the relative financial well-being of the consumer. The consumer may prefer B to A, but because it is too expensive he uses A. As his income increases he will be able to use B and as it increases more he may use C.[1] The goods which are used at the higher income levels are referred to as superior, those at the lower levels inferior. If the hierarchy is considered in its complete amplitude, it may be extended almost indefinitely. We may consider different qualities of bread, then go on to different qualities of meat, drinks, etc., each satisfying hunger but varying, in so far

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[1]Pareto recognizes here the problem of inferior goods which we discussed previously. He does not attack this with the use of indifference curves, but in his discussion he does show how the increase in income will cause less of the inferior good to be consumed. Our discussion below went further than that. It showed the drop in quantity consumed of the inferior good as a result of the drop of price of that good; that is, the drop in price set up an income effect. See Manuel d'Economie Politique, pp. 258, 274. Below pp. 56-57.

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as they are substituted, as income changes. As was the case in complementarity, the concept here must be limited. Either we can consider only those cases where the effects are very important, or we can take pairs of substitutes and consider the others as given. But no matter how it is done the concept must be limited so that only those elements will be considered which will make up a useful theory.

In almost every economic problem the element of time is a factor. Pareto recognizes this with regard to the problem of related goods and he attempts to treat it. However, his treatment adds nothing to the development of a dynamic system and his theory remains essentially static. He shows how changing conditions over a period of time bring about changes in tastes. To be very exact any change considered in relation to time is a radical one. "An individual on the morrow is not the same as he was the day before."[1] But for economic purposes this Heraclitian approach is not useful for one need not be so exact. It is sufficient to divide the changes in time into the long run and short run and consider the effects. In the long run one cannot compare the new curves with the old, because there will have been a change which has come about. The long run infers this change. The short run on the other hand excludes any change so that the indifference curves remain the same. This technique is of course not new and to the extent

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[1]*Ibid.*, p. 260.

that it is so used by Pareto one can probably agree with him. However, it is very possible that one may disagree with his notion of the length of time involved in the short run. He suggests for the group curve a period of two or even four or five years, the idea being that in the long run changes must have sufficient time to work themselves out. His example is an assumed change in taste by the Italian people in their consumption of coffee and tea. A change of this sort may perhaps require five years to come about, and in the light of the above technique any period less would have to be considered as a short run. However, there is an element of gradual change which enters, and this I feel is important, especially since the individual indifference curves are almost instantaneous. A period of five years is rather long for a short run period. However, as this technique is used it would have to be so considered.

Aware now of the ramifications of the concepts and their limitations, Pareto proceeds to examine them in relation to marginal utility analysis. For this purpose he makes a very necessary assumption;namely, that the individual always knows whether the change in utility due to the transition from combination I to II is greater or less than the change from combination II to III. This assumption is highly significant for in Pareto's mind it allowed the comparison of marginal utilities on an objective scale of preferences with utility

considered as a quantity only. The assumption does not say by how much one transition is greater or less than another; it merely assumes that the individual knows which he prefers. It will be remembered from the development of the curves that the slope of the tangent to an indifference curve at a given point indicated the marginal utility of one good at that point. Now with the assumed curves Pareto still needed some device which would give meaning to marginal utility and yet not have the utilitarian aspects of marginal utility. This was accomplished, so he thought, by the above assumption, because by it Pareto could still refer to marginal utility without bringing in any direct measurement.[1] It accomplished what he was seeking.

Having laid the necessary foundation Pareto now established his definitions of the different aspects of related goods. He considers first an increase in B, A remaining the same. If A and B are complementary an increase in B, A remaining the same, will generally bring about a decrease in the marginal utility of B. Also if they are substitutes,an increase in B, A the same, will decrease the marginal utility of B. To this point he is merely repeating the law of diminishing marginal utility. But he then asks, what happens

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[1]It is at this point that Pareto compromises his objectivity,for actually this assumption indirectly admits the measurability of utility. See above pages 39-42.

to the marginal utility of A when B is increased? And from the answers to this question he derives his definitions of substitution and complementarity. For goods that are complementary an increase in the quantity of B will generally increase the marginal utility of A. The pleasure derived from one lamp added to others is greater the more oil one has. Thus he defines complementarity such that an increase in B raises the marginal utility of A. For substitutable goods the relationship differs. If A can be substituted for B, then the more B one has the less will be the marginal utility of A. He therefore defines substitute goods such that an increase in B decreases the marginal utility of A. These are his definitions.

But immediately there arises the question of proof. From the first assumption Pareto was able to treat variations in marginal utilities objectively. Now that he has framed his definitions in terms of these fluctuations, it becomes necessary to examine them on the indifference map to see just how they apply, and there should follow from the indifference map a geometrical and economic proof. Before we do this, however, it should be pointed out that by formulating the definitions in terms of fluctuations of marginal utilities, Pareta has adopted an approach which considers the two cases of dependency as extremes on a scale. An increase in B brings about either an increase or decrease in the marginal utility of A, and the

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condition of dependency is governed by that fluctuation. This immediately sets up a scale on which one can place different degrees of dependency. Thus when we examine the definitions in the light of the indifference map the scale of degrees should also be present.

For the proof on the indifference map Pareto begins with the situation where the individual possesses two goods A and B, but of these only A is useful to the individual. This is admittedly a peculiar case, but it is entirely possible and there is nothing wrong in using it as a starting point. It is not a case of independent goods. The marginal utilities of the two goods are dependent on each other, but one is a constant, zero. Now, picturing this situation on the indifference map, the hill of utility will rise from the B axis, the height increasing at a decreasing rate. The hill will be in the form of a cylindrical surface any section of which parallel to the A axis will be indicated by the shaded area in Diagram 16. Since the utility of B is zero the indifference curves will be straight lines parallel to the B axis. For a given amount of A in combination with any amount of B this shows the constant utility required of the indifference curve. Furthermore as the quantity of A decreases, in combination with any quantity of B, its marginal utility decreases. This follows from the law of decreasing marginal utility and it is indicated on the utility surface by the decreasing slope of the surface from

c to b". These so far are basic properties indicated by the utility surface which follow from previous discussion.

In the next step Pareto examines the line of exchanges. It will be remembered that this is formed by joining the points of equilibrium, that is, the points of tangency between the "sentiers" and the indifference curves. The situation here by hypothecation is peculiar. The individual possesses B which is not useful to him; he will, therefore, never demand it, but he can always offer it. Thus the line of contracts is different than that which we met before. If possessing various quantities of A the individual never demands B, then that line which indicates the various quantities of A and no B must make up part of the line of contracts, for every point on it is for the individual a point of equilibrium. Furthermore since B is always given up and never demanded, its demand price is zero. The price of A on the other hand depends on the position of the curves a b, a'b', etc., and any point on a given indifference curve, becomes a point of equilibrium. The line of contracts then, depending on the price of A, is made up of the A axis and the indifference curve.

We now come to another definition which accounts for the peculiar situation from which the proof is started. Pareto now defines complementary goods in the perfect case. Here A and B are complementary only when they can be used in combinations vigorously defined. The definition here in the

perfect case is apparently different from the one given previously. It is now based on quantity rather than marginal utility. Actually the two definitions are the same,for although Pareto does not show it, the definition based on marginal utility does in the perfect case reduce quantitativity to goods used in vigorously defined proportions. This will be apparent from the proof. But on the basis of this definition perfectly complementary goods are those dependent such they can be enjoyed only in combinations vigorously defined. The only quantities demanded will be those which meet this requirement. The others, though possessed, will not be demanded but they can be offered. On the indifference map the curves in this case will be represented by straight lines which cut each other at right angles (Diagram 17). The pleasure which an individual derives on any indifference curve will be the same. However, while different quantitative combinations of A and B are possessed, it is only at c that the necessary vigorously defined requirement is met precisely. It is met also at all other points on the curve, but at these points there is an excess of one good or the other, and being an excess it is offered and not demanded. At c the requirement is met exactly. We see therefore why Pareto started from the peculiar case where goods are dependent but only one good is useful. At every point on the perfectly complementary curve, except at c, this situation prevails to some extent. Of

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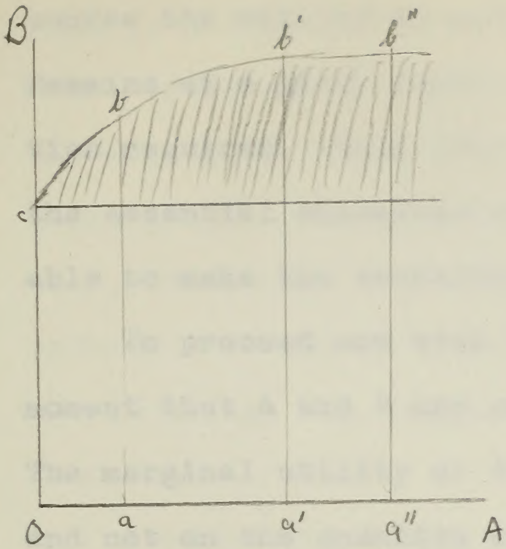


DIAGRAM 16

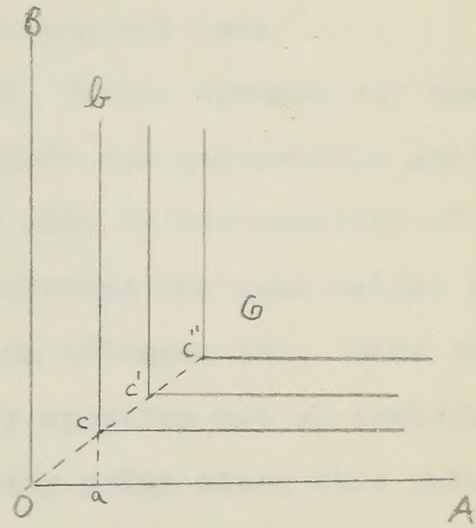


DIAGRAM 17

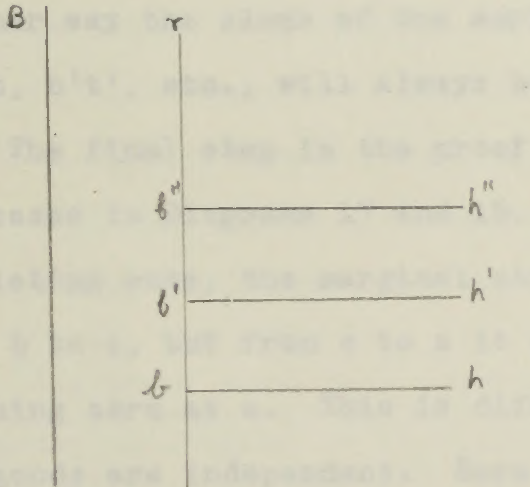
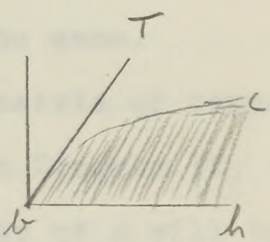


DIAGRAM 18



course the utility of one good is not always zero, but it remains at a given constant according to the defined proportion required. Thus from this peculiar case which contains the essential characteristics of the perfect case, he is able to make the transition to the perfect case.

To proceed now with the proof, Pareto assumes for the moment that A and B are not dependent but independent goods. The marginal utility of A depends only on the quantity of A and not on the quantity of B. Following the same method he erects a surface for this situation (Diagram 18). Here the indifference curve (it is strictly speaking not an indifference curve) is represented by ur. At any point along this curve b, b', b", the marginal utility of A will be the same, because the utility of A is independent of that of B. Expressing this another way the slope of the surface at points b, b', b", or bt, b't', etc., will always be the same.

The final step in the proof consists of comparing the two cases in Diagrams 17 and 18. In Diagram 17, the perfect completing case, the marginal utility of A will be the same from b to c, but from c to a it diminishes very quickly becoming zero at a. This is different in Diagram 18 where the goods are independent. Here the marginal utility of A remains the same all along the curve. It must necessarily follow then that Diagram 17 represents completing goods. By itself it is the sum of two different dependencies of the

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type indicated in Diagram 16. But since each dependency taken by itself is not an independent relationship of the type in Diagram 18, then it must be a dependent relationship; and since at point c it meets the vigorously defined proportion then, it follows that the curve indicates the perfect complementary situation.[1] Thus by a rather intricate route Pareto has indicated the nature of the indifference curve in one perfect case.

Fortunately, the other limit involves much less difficulty. Here again Pareto does not utilize his definition based on marginal utility, but resorts to a quantitative relationship. A and B are perfect substitutes when a quantity of one can replace a quantity of the other in exactly the same proportion; for example, A and B are perfect substitutes if 4 of A can substitute for 3 of B, 8 of A for 6 of B, etc. As in the case of complementarity, this definition follows from the previous one, although Pareto has not bothered to show how it does in his context. On the basis of marginal utility the previous definition for the perfect substitutable case

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[1]The proof here is a negative one. Since the slope of the surface increases rapidly from c to a, it follows that the goods in Diagram 17 represent dependent and not independent goods. It might be objected here that the fact that goods are not independent does not prove that they are dependent. However, it will be remembered that the relationships are classified by definition as dependent and independent only, and if they are not independent, then it must follow from the classification that they are dependent. See Manuel d'Economie Politique, p. 279.

becomes: A is perfectly substitutable for B when an increase in the quantity of B decreases the marginal utility of A in the exact proportion that that of B has been increased. This can be expressed quantitatively as we have done above.

Now translating this definition on to the indifference map, it is found that in the perfect case, the curves become straight lines parallel to each other; and their slope is determined by the ratio in which they substitute for each other. This follows directly from the definition, and without further proof it becomes possible to indicate the nature of the curves in the perfect case. It remains to point out incidentally that in the position of equilibrium in the perfect case, the line of contracts, or "sentier," will coincide with the indifference curve.

To this point Pareto has set up definitions of the perfect cases at either end of the scale and he has indicated the nature of the indifference curves in each case. The question which follows is what happens in between these limits, and here unfortunately Pareto is weak. His reasoning is merely a glossing over of the situation. If the L-shaped curve indicates perfect complementarity then it is this characteristic of the curve which indicates complementarity to a greater or lesser degree. Thus the more bent are curves the more do they indicate complementary goods, the less bent, the less do they indicate complementary goods. Similarly for perfect

substitution, the essential characteristic of the curve is flatness; so the more flat the curves are the more substitutable are the goods, the less flat the less substitutable.

It is at once obvious that Pareto's work is lacking. He has set up limits and to a reasonable degree has succeeded in coinciding the indifference curves with his definitions in these limits, but the area in between is sadly lacking. It is not sufficient to glide over this area with a "more or less" reasoning, particularly since it is in this area that the greatest actual number of relationships occur. While the perfect case is necessary to the theory, in actuality it is the exception. Pareto's theory is not wrong, but rather inadequate. However it has established certain parallels between his definitions and the indifference curves in the limiting cases, and to this extent it adds something.

Allen

Allen recognized the difficulties inherent in Pareto's treatment and his work was an attempt to correct them.[1] His approach was somewhat different. Whereas Pareto first established limits to a scale and tried to coincide the intervening positions on the basis of these limits, Allen starts from the middle of the scale and attempts to establish the positions

[1]The substance of this section is based on Allen's article, "The Nature of Indifference Curves" in Review of Economic Studies, February, 1934. This work was done before Allen's collaboration with Hicks and is therefore to be considered as an independent contribution.

on each side. In so far as he utilizes the indifference curve, Allen relies on Pareto's previous development. It is only in his discussion of related goods that his approach differs, thereby enabling him to add something new.

Allen considers first the case of completely independent goods. This occurs when two goods X and Y each make "separate and independent" contributions to the utility function so that the total utility function appears as the sum of two separate and distinct functions. This is somewhat different from Pareto's use of the word and is therefore to be noted. As in Pareto's sense the goods here are not related; that is, there is no relation whatever between the marginal utilities of X and Y. For each good by itself the law of decreasing marginal utility applies. But whereas Pareto considers each good by itself and arrives at the perfectly straight indifference curve of Diagram 18, Allen considers the two goods together so that for a particular combination the total utility of the two goods is made up of the sum of the separate utilities of each, or the total marginal utility of the combination is made up of the sum of the separate marginal utilities of each.

To attain his ends Allen requires two basic assumptions. He assumes first that the second derivative of each utility function is always negative; that is, that the marginal utility of each good decreases as the quantity of that good

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To attain his ends Allen requires two basic assumptions. He assumes first that the second derivative of each utility function is always negative; that is, that the marginal utility of each good decreases as the quantity of that good

increases, the law of decreasing marginal utility. Secondly he assumes that the marginal utility of X becomes zero at a definite saturation value of X and that this value is related to Y so that at that point it is a function of Y. In other words the marginal utility of X varies from a positive value to zero to a negative value. Similarly the marginal utility of Y becomes zero at a definite saturation value of Y such that at that point it is functionally related to X. Now strictly speaking these assumptions are not necessary, "but they are appropriate to a 'normal' type of individual behavior."[1] As Allen explains, it is quite possible to assume that the marginal utility of X and Y increase as their quantities increase, but for the sake of convenience it is better to use the opposite assumption since it is required that certain portions of the indifference curves be convex to the origin, a fact which follows on the above assumption.

On the basis of these assumptions Allen examines the utility surface in the case of independent goods. Utilizing the three dimensional device the utility surface appears in the form of a hill. Those sections perpendicular to the utility ordinate and parallel to the base will be the contours of the hill recognizable as indifference curves. But here special attention is given to the vertical planes, those

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[1] Ibid., Appendix, p. 118.

perpendicular to OX or OY. It is assumed that a constant value is given to Y; then in this case the utility becomes a function of X only. On the utility hill this will appear as the section of the hill made by the plane perpendicular to OY at the given value for Y. This section of the surface will show the variation in utility height corresponding to various amounts of X at the given amount of Y. As the given value of Y changes a series of such sections will be obtained each of the same form but varying in height. On any section, the slope of the tangent at any point on the surface is the marginal utility of X. (Since utility depends on the amount of X only for a given value of Y, it is a function of X only and the marginal utility is indicated by the first derivative of that function or the slope of the tangent to the surface.) Now on the basis of the assumptions made above each curved section will rise at a decreasing rate up to the saturation value of X after which it falls. Each section then has some maximum value for X, the saturation value where the marginal utility changes from positive to negative. Similarly considering the vertical planes perpendicular to OX for given constant values of X, there will be a series of saturation values for Y. Now joing these saturation values to form a continuous line, the utility surface appears as a hill with two intersecting "ridge lines." As we shall see these ridge lines are important for they represent a new aspect of the

indifference tool which will allow for further insight into the problem. Each ridge line by itself consists of the highest points of the sections perpendicular to it and thus gives the partial saturation point for each good separately. Every point on the ridge line is a point of complete saturation for one good, where the marginal utility for that good is zero; then the point of intersection of the ridge lines is the point of complete saturation for both goods, the peak of the hill.

Reverting now to the two dimensional system of indifference curves, Allen introduces on these curves the new idea of ridge lines. The indifference system appears now as in Diagram 19. It shows the indifference curves in relation to the complete saturation point c, with the partial saturation lines AA' and BB' intersecting at c. The OXY plane is divided by these lines into 4 areas, the characteristic of each being the sign of the marginal utility function of each good.

It can be shown from the assumptions made above that in three of these regions at least one of the signs of the marginal utility of X or Y will be negative. However, in the rectangular area OBCA both are positive. This area Allen calls the "effective" region. It is in this region that the indifference curves are always negatively sloped and convex to the origin. Any point moving parallel to the OX axis will cross indifference curves in an increasingly higher order of utility up to the line AA'. Beyond AA' the order of utility decreases.

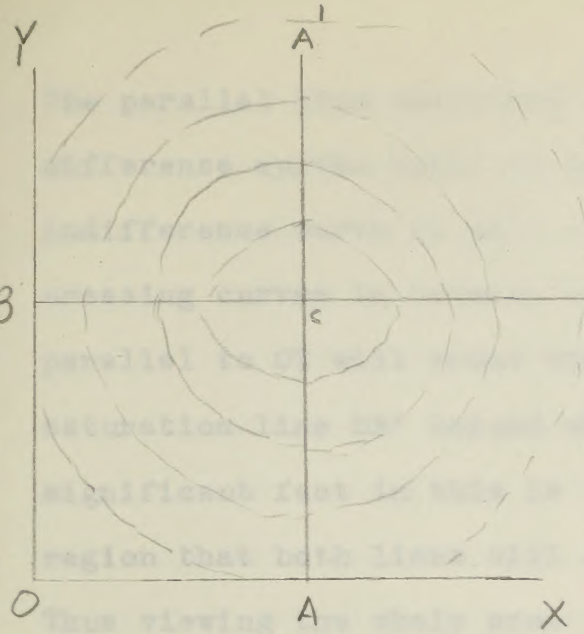


DIAGRAM 19

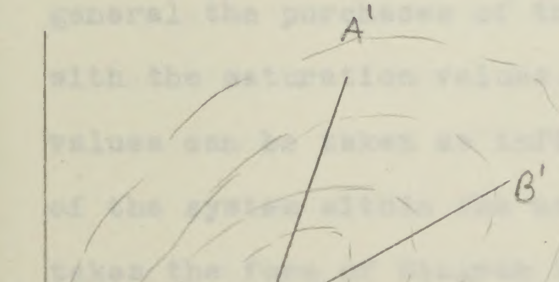


DIAGRAM 21

DIAGRAM 20

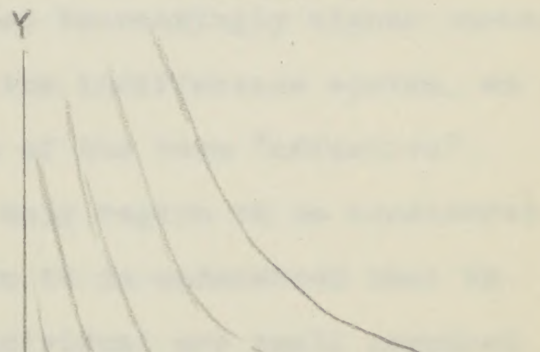
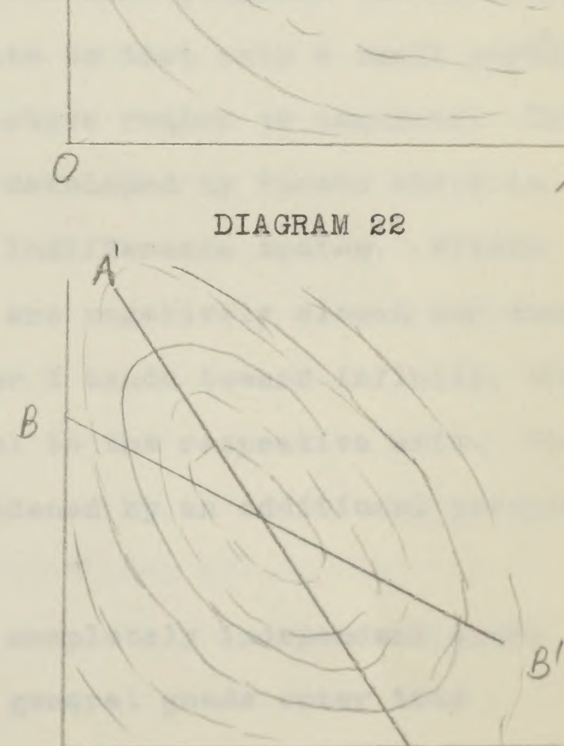


DIAGRAM 22



The parallel line described by this point crosses higher indifference curves until it becomes tangent to the highest indifference curve at AA'. Beyond AA' the line crosses decreasing curves in reverse order. Similarly a point moving parallel to OY will cross higher indifference curves to the saturation line BB' beyond which the curves decrease. The significant fact in this is that it is only in the effective region that both lines will cross increasingly higher curves. Thus viewing the whole area of the indifference system, we see the validity of Allen's use of the term "effective" region, for this is really the only region to be considered. This is brought out further when it is understood that in general the purchases of the individual are small compared with the saturation values. For most purposes the saturation values can be taken as infinite so that only a small portion of the system within the effective region is required. This takes the form of Diagram 20 developed by Pareto which is the conventional form of the indifference system. Within this partial area the curves are negatively sloped and convex to the origin. As either X or Y tends toward infinity, the curves tend to become parallel to the respective axis. Thus the conventional form is broadened by an additional perspective.

So far only the case of completely independent goods has been considered. But in general goods enter into

consumption in various related ways so that usually the marginal utility of one is directly dependent on that of another (or of several others). Considering two goods X and Y, the marginal utility of X and that of Y depends now on the amounts of both X and Y possessed. The utility function is no longer a mere summation of the marginal utilities of each but rather an expression containing values of both, the variations in which are obtained only by using the partial derivative form. Under this situation, saturation values for both goods exist, as they did before, but they are not constant values throughout; that is, the ridge lines are not perpendicular to the axes. The value of X which makes the marginal utility of X vanish will depend on a constant value of Y in the partial derivative, but this will be a varying constant value; that is, the ridge lines will take different directions depending on the way the goods are related. Under this method then the relationships are regarded more readily according to the twisting and turning of the ridge lines. There are, however, certain essential characteristics which are still preserved. The concept of the effective region still applies. Within this region the indifference curves are negatively sloped and convex to the origin. Also the indifference curves increase in higher order the closer they get to the complete saturation point c. Finally at all points where the indifference curves cut the ridge lines, tangents at these points

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will be parallel to the axes. For the exact form the ridge lines and indifference curves take in every case it is necessary to know the exact nature of the relationship between two goods. Allen does not attempt to examine all possibilities. However, he does indicate the two cases which are typical of the two general classes of related goods.

In the first case the marginal utility of XY, ϕ_{xy}, is positive at all points.[1] The marginal utility of X increases as the amount of Y increases and the goods are complementary. Here the ridge lines bounding the effective region are positively sloped so that the saturation value of X is greater the greater the amount of Y possessed, and conversely. This is diagramed in Diagram 21. It will be noted that within the effective region the basic elements of the indifference system, previously attained, will still apply. However, the variation in the slope of the curves within the effective region will differ. In this case the slope becomes flatter as the tangent to the curve moves parallel to the X axis and steeper as it moves parallel to the Y axis. The significant characteristic of the relationship is indicated thus not only by the direction of the ridge lines but also by the shape of the curves within the effective region, as shown by the variation in the slope of the tangent to the curves.

In the other case, Diagram 22, the goods are competitive. Here the marginal utility of XY, ϕ_{xy} is negative at all points,

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[1] ϕ_{xy} is the second derivative of the marginal utility function.

and the marginal utility of X decreases as the amount possessed of Y increases. The ridge lines bounding the effective region are negatively sloped since the saturation value of X is greater the smaller the amount of Y possessed. The essential form of the system is the same as in the case above with the exception that the ridge lines are negatively sloped and the shape of the curves is different. The slope of the tangent to the curves will be flatter as the point moves parallel to the X axis and steeper as it moves parallel to the Y axis, but the change in the slope is not as sudden or in other words the curve itself is flatter. Allen has thus established two criteria to express the nature of the relationship. The first is the sign of the second derivative, ϕ_{xy}, the second is the direction of the ridge lines.[1] But for the most part these criteria are similar to those of Pareto. The point in question is how they differ.

It will be remembered that Pareto's definitions were based on the variations of marginal utilities. By framing his definitions in these terms Pareto assumed the existence of a total utility function. As was previously pointed out, Pareto compromised at this point on the question of measurability. He evidently overlooked the fact that the marginal

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[1]It is to be noted that the ridge lines need not be straight lines as shown in the figures. They can be curves, but they must lie in directions which will form either an acute or obtuse angle, as is called for by the nature of the relationship.

utility expression becomes indeterminate because it depends solely on the existence of a total utility function which itself is indeterminate if the indifference curves are assumed as given, the way he does. Allen recognized this and attempted to construct his definitions so that the criteria of complementarity and substitution would be independent of the existence of a utility function and the indeterminateness of that function.[1] He does this by elaborating on an idea first developed by Johnson.[2] Instead of assuming indifference curves as Pareto did, Allen assumes continuous marginal utility functions. On the bases of these functions the scale of preferences or indifference system can be represented by ratios of marginal utilities so that in the relationship of any two goods the expression $-\frac{\phi_x}{\phi_y}$ will indicate the direction of preference. In this way he arrives at the same indifference curve and can still use the variation of the slope to explain the relationship between goods. This is a definite theoretical improvement over the work of Pareto.

In so far as he is theoretically more precise, Allen improves over Pareto. But essentially he adds nothing to the solution of the central problem, for the same difficulties

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[1]See R. G. D. Allen, "A Comparison Between Different Definitions of Complementary and Competitive Goods," <u>Econometrica</u>, April, 1934, pp. 168-175.

[2]Johnson, <u>op</u>. <u>cit</u>., pp. 495-496.

which existed after Pareto's work are still present. Even with the use of ridge lines it is impossible to coincide a particular relationship with a particular shape of indifference curve or slope of the ridge lines. The degree of shading from one case to the other and the particular degree still present a problem. Allen has explored certain ramifications of the indifference system and in so doing has given additional understanding to the meaning of the concepts involved in the problem of related goods, but on the whole his work is not too far advanced over that of Pareto.

On the question of measurability of utility it should be clearly understood that, although Allen's assumption was more precise than Pareto's, he still did not avoid the use of the immeasurable concept. Marginal utility, whether it is assumed in the form of ratios of continuous functions or merely derived from a total utility function, cannot be measured, for it involves a type of introspective data which is incapable of objective expression. In terms of the two basic assumptions discussed at the end of Chapter II, it assumes that the individual can compare gains between transitions from different combinations. Such gains, of course, aside from the fact that they can be expressed only through introspection, are meaningless, for the individual in maximizing his position will always seek the highest combination, however small the gain may be. Allen attempted to avoid this pitfall,

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but it is quite evident that at this stage he was not fully aware of the psychological implication which his new assumption entailed. However, the difficulty of measurement (mathematically the determinateness of the second order derivative) was not what held back further development in the field. To be sure this was vital to the problem, but it was not the key. The light which led to the Hicksian development came from the reworking of the indifference analysis on the basis of new definitions which did not include marginal utility, but the real key to the Hicksian development (which was recognized by Allen and Hicks in collaboration) came from a fuller understanding of the relationships of the concepts of complementarity and substitutability, particularly in so far as money entered. We shall examine this in the following section.

Hicks on Complementarity

The traditional treatment of related goods has been in terms of utility, but as we saw in Chapter III, Hicks did away with the utilitarian concepts and replaced them with concepts of substitution which were similar in purpose. But having done away with fluctuating marginal utilities, he also disposed of the criteria for substitutability and complementarity. If he had replaced fluctuations in marginal utilities by fluctuations in marginal rates of substitution, he could perhaps treat the case of two goods where one substitutes

for the other, but the moment he attempts to treat complementarity this breaks down, because the fluctuations of the marginal rate of substitution cannot explain complementarity unless it is known what each good substitutes for. Under the new concepts a common denominator is clearly necessary. This is found in money, whatever its form. It may be in the form of legal tender as is customary or it may be in the form of some third good which serves as a yardstick. But whatever the form there must be a third comparable.

The necessity for the third good which serves as a basis of comparison is a direct result of the changing of diminshing marginal utility into diminishing marginal rate of substitution. Hicks wanted a definition of substitute goods which would make absolutely certain that an extra unit of the same physical commodity was a substitute for preceding units. The definition of substitute goods on the basis of fluctuating marginal utilities did not afford this certainty of unitary substitution because the marginal utility of money was not considered. (It was assumed to be constant.) But by using the third good as a denominator of comparison the marginal utility of money is considered. It is definite then that an additional unit of X will lower the marginal rate of substitution of X for money; that is, the additional unit of X will be a definite substitute for the preceding units. Considering X and Y (money) on the indifference map, an

additional unit of X definitely lowers the marginal rate of substitution of X for money if X is substituted for money in such a way as to leave the consumer no better off than before. This results from the law of diminishing marginal rate of substitution (between X and money). Therefore in defining substitute goods it readily follows _that Y is a substitute for X if the marginal rate of substitution of Y for money is diminished when X is substituted for money in such a way as to leave the consumer no better off than before_; for if substituting X for money lowers the marginal rate of substitution of X for money, and if substituting Y for money lowers the marginal rate of substitution of Y for money, then, if by the definition X and Y are substitutes, when the marginal rate of substitution of one is lowered it must induce a lowering of the marginal rate of substitution of the other. This follows from the principle of diminishing marginal rate of substitution and the use of the third good as a medium of comparison. For complementary goods the definition follows along the same lines, but in a different direction. _Y is complementary with X if the substitution of X for money increases the marginal rate of substitution of Y for money in such a way as to leave the consumer no better off than before_. Here the substitution of X for money induces a change in the marginal rate of substitution of Y for money, but in the opposite direction.

These definitions are complicated, but when digested it will be seen that they clarify the meaning of two phases of related goods. In comparison to the previous definitions they certainly are free from quantitative utility measurement. Yet they can be tied together. The Edgeworth-Pareto definition was based on the assumption that the marginal utility of money was constant. Treating the effect of the change in marginal utility of one good on another, it was necessary to make this assumption, or to infer it, in order to rule out the effect of any change in the marginal utility of money.[1] Therefore on the new definitions, if it is assumed that the marginal utility of money is constant, that is, no income effects will be wrought by changes in the marginal utility of money, then they will reduce to the previous definitions. But it is one of the improvements of the new definitions that they can be applied in cases where the marginal utility of money cannot be assumed to be constant.

The new definitions are more easily understood if they are translated into Marshallian terms. From our early discussion it will be remembered that the marginal rate of substitution of X for Y is in effect what Marshall calls the marginal utility of X in terms of Y.[2] Transcribing this into the present

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terminology, if we let Y be money, the marginal utility of X in terms of money is equal to price, which in Hicksian terminology is equal to the marginal rate of substitution of X commodity for money.[1] This will help in understanding the new definitions, for they take on a clearer meaning in Marshallian terms. If in the previous definitions we now put the word "price" in place of "marginal rate of substitution of money" they become: Y is a substitute for X if the (demand) price of Y is diminished when X is substituted for money in such a way as to leave the consumer no better off than before. Y is complementary with X if the (demand) price of Y is raised when X is substituted for money. Clearly then the idea is that in the case of substitute goods an increase in X lowers its price so that more of X is demanded. As income is fixed this causes less Y to be demanded so that the price of Y drops. With complementary goods the increase in X lowers its price, allowing more of it to be purchased. This in turn induces an increase in the demand for Y thereby raising its price. In Marshallian terms therefore the difference between the two cases depends on whether the increase of X which accompanies its drop in price induces an increase or a decrease in the price of Y and also the quantity of it demanded. But all this is based on the premise that the

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[1]Hicks, Value and Capital, op. cit., p. 20.

marginal utility of money is constant. If we remove this premise, then the marginal utility of X in terms of money is not equal to the price (income effects now enter) and Hicks' more complicated definitions become necessary.

Having established new definitions it now is necessary to see what kind of a theory can be derived from them. And oddly enough there are some peculiar consequences. It will be remembered that one of the immediate results of Pareto's development of the indifference analysis was that it served to throw some light on the problem of related goods. Now, however, on the basis of the new definitions, the indifference tool is of little use. By its very nature it is useful only when the consumer is assumed to spend his income on two commodities. This means that it is applied generally to treat the demand for one commodity and all others (money). For this type of problem the indifference analysis is very informative. It allows a more thorough explanation than Marshall's method since it also includes income effects. But for the problem of related goods it fails almost completely for this problem cannot be treated on the two dimensional indifference map. It needs three dimensions to represent the three goods, X, Y, and money, the common denominator of exchange. Since the tool which formerly facilitated explanation now does not apply, the theory must be represented in ordinary words.[1]

[1]It can be represented in algebra. See ibid., Appendix, Chapter 3.

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In the explanation of the income and substitution effects set out in the discussion of the theory of demand, it was seen how the effects set up by a fall in the price of a given good, exert themselves on the demand for that good. Now it is necessary to see how they work out in the general rearrangement of the consumer's expenditure, particularly in the expenditure on those goods which are related in consumption. First let us repeat the results previously attained.

A fall in the price of X reacts on the demand for X and for other goods through two effects, the income effect and the substitution effect. The income effect treats the conditions where the fall in the price of X acts like an increase in income. It tends to increase the demand for every good consumed, except inferior goods. If the proportion of income spent on X is small, the income effect will be small and will have little influence on the demand for X or for other goods. The substitution effect, on the other hand, involves a substitution in favor of X and against other goods. If all other goods are lumped together and treated as a composite commodity, the substitution effect tends to decrease the demand for the composite commodity. It does not decrease the demand for each good taken separately but it does decrease the demand for all goods lumped together.

Now if, according to the new definition of complementarity, X is complementary with Y, one good out of the composite, then

it is known that if Y is held constant, a substitution in favor of X and against money (a decrease in the marginal rate of substitution of X for money or a fall in the price of X) will raise the marginal rate of substitution of Y for money. This follows right from the definition itself. But the price of Y is assumed constant, therefore an increase in the marginal rate of substitution of Y for money tends to encourage a substitution of Y for money so that the marginal rate of substitution of Y for money is kept equal to its price.[1] In other words a fall in the price of X tends to encourage an increase in the price of Y or in terms of the present definitions, if Y is complementary with X, a substitution in favor of X tends to be accompanied by a similar substitution in favor of Y.[2]

But if Y is a substitute for X, then according to the definition a substitution in favor of X and against money (Y constant) tends to encourage a substitution against Y and in favor of money. In other words the substitution in favor of X induces a substitution against Y.

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[1]The marginal rate of substitution between X and Y is proportional to the prices of X and Y, but the marginal rate of substitution of X and money is equal to the price of X.

[2]If the price of Y is kept constant, the decrease in the marginal rate of substitution of X for money lowering its price cannot raise the price of Y but it does increase the marginal rate of substitution of Y for money which, when Y is allowed to vary, will tend to raise its price. The marginal rate of substitution of a good for money and the price of that good are not the same thing. When the marginal utility of money is assumed constant as in the Marshallian analysis, then they are the same. But Hicks makes no such assumption so that it is possible to hold the price of Y constant and allow the marginal rate of substitution of Y for money to vary.

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Now what is the connection between this type of substitution, that which is opposed to complementarity, and that which was discussed earlier? The distinction between the two cases, when made as above, clears this up, for it can now be said that they are the same. When a consumer divides his income between two goods only, the purchase of all others being ruled out, then a substitution relation is the only relation that can exist. This applies above with regard to the marginal rate of substitution of a good and money. If the consumer is to have more of one and still be no better off than before, then he must have less of the other. This works out particularly well on the indifference map. But when income is divided between more than two goods other relationships are possible. Let us assume three goods X, Y and Z with Y and Z both substitutes for X. An increase in X can only be made against Y and Z, if the requisities of the problem are to be maintained; namely, that the consumer is left no better off than before and that the marginal rates of substitution between other goods remain unchanged. But this substitution is made against each good taken separately. Yet it is quite possible that for the two requisites to be satisfied, one of these goods must be increased. This occurs if either Y or Z is complementary with X. In this case, however, the increase in Y, if Y is complementary with X, will be made at the expense of Z, for Y and Z taken together as composite must

decrease. Thus it is seen why complementarity does not arise on the indifference map. X and Y can only be complementary if there is some third good at whose expense the substitution of both X and Y takes place, and the indifference map, since it is a two dimensional device, cannot show this third good. Obviously then, the essence of indifference analysis is substitution, the same type of substitution which is defined in our definition above. Indeed it is the definitions of complementarity and substitution, as opposed to it, which bring this out and which tie together the two types of substitution.

The key to the Hicksian treatment of related goods lies in the recognition that complementarity is possible only at the expense of a third good and also in the introduction of money as the good at whose expense substitution takes place. Considering three goods X, Y, and money, if X and Y are complementary there must be a third good against which substitution takes place. If X and money substitute, then Y and money must also substitute. Of four goods, X-Y-Z and money, X, Y, and Z may be complementary, but if they are, each must be a substitute for money. Thus no matter how many goods enter into the consumer's expenditure, if they are all complementary, they all must substitute for money, or for some outside good. This establishes a theroetical maximum limit of complementarity. Of n goods, all may be complementary except one. The limit of complementarity then is n-1 goods. The other limit

is a complete lack of complementarity. Of course in actuality the most common cases will be those which come near the minimum of complementarity. One particular good will usually have a small group of goods closely associated with it in a complementary relationship, but with regard to another good chosen at random the most likely relationship to exist is that of mild substitutability.

Hicks--Theory of Related Goods

Having seen the definitions established by Hicks and the consequences of these definitions, it remains now to sum up the conclusions into a consistent and precise theory. In the theory of demand the investigation was made in the light of the effect of a change in the price of commodity X on the consumer's expenditure. The same procedure will again be followed.

A fall in the price of X affects the demand for X and the demand for other goods through the substitution effect and the income effect. As far as the demand for X is concerned the substitution effect increases it; the income effect diminishes it unless X is an inferior good. As for the demand for other goods taken compositely, the substitution effect diminishes it, the income effect increases it. Whether or not the demand for other goods will move in one direction or another depends solely on the magnitude of each effect and the

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extent to which one counterbalances the other. All this merely follows from results attained through the use of the indifference system.

Now with regard to the effect of a fall in the price of X on the demand for Y, some particular good in the composite commodity, the direct dependence on the indifference analysis fails, for here we must consider whether Y is complementary or substitutable with X. Ordinarily the fall in the price of X sets up a substitution effect which diminishes the demand for Y, but this does not necessarily follow, for if Y is complementary with X, the substitution effect induces as similar substitution in favor of Y increasing the demand for Y. As far as the income effect set up by a fall in the price of X is concerned, regardless of the relationship it will increase the demand for Y, unless of course Y is an inferior good. Thus in considering all the influences in the demand for Y, not only must there be included the income and substitution effects, but also the relationship between the goods themselves in so far as this alters the substitution and income effects. It is possible to distinguish several cases within which various possibilities may occur.

1. Y may be highly complementary with X. A fall in the price of X may induce an income effect or substitution effect on the demand for X. Both effects move in the same direction but if the substitution effect on the demand for X is large

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I. Y may be highly complementary with X. A fall in the price of X may induce an income effect or substitution effect on the demand for X. Both effects move in the same direction but if the substitution effect on the demand for X is large

relative to the income effect, then it may completely outweigh it so that the induced demand for Y will move very closely with the demand for X. An example of this, and Hicks specifically points out that this is only an example, is the case where X and Y must be used in fixed proportions so that the substitution in favor of X is matched by the substitution in favor of Y. There may be other cases where the substitution effect on X drowns out the income effect, but aside from the proportion in which the goods are used, when this occurs the demand for Y will definitely increase. When the goods are highly complementary this situation usually prevails because by the very nature of the relationship the income effect is of little importance.

2. Y may be mildly complementary with X. Here the income effect becomes more important. Usually in its reaction on the demand for Y, it goes in the same direction as the substitution effect so that total effect corresponds to an increase in Y. But if Y is an inferior good the two effects oppose each other and may cancel out. In the very extreme case the income effect may outweigh the substitution effect so that the demand for Y will diminish slightly.

3. Y may be mildly substitutable for X. This is a very common situation. Ordinarily the income and substitution effects in this case will go in opposite directions, leaving only a very slight effect on the demand for Y which may go

in either direction. However, if Y is an inferior good the demand for it will definitely contract, though perhaps very insignificantly.

4. Y may be highly substitutable for X. Here the substitution effect will dominate and the demand for Y must diminish. Also here it is possible to designate the extreme case where X and Y are perfect substitutes. This occurs when a substitution in favor of X reducing the marginal rate of substitution of X for money induces a reduction in the marginal rate of substitution for Y for money by exactly the same proportion as that in which that of X has been reduced. This cannot be illustrated on the indifference map since the definition is in terms of money (the third good), but if we revert back to the earlier terminology, this will occur when the marginal rate of substitution of X for Y decreases in exactly the same proportion as that in which the marginal rate of substitution of Y for X increases. On the indifference map this will appear as a series of straight lines.[1] For a precise theory, however, it is necessary to stick to the definitions and since money is required in the definitions it is incorrect, strictly speaking, to refer to the indifference map. In his explanation Hicks does not make this reference, but I bring it in here to make the point that on the basis of the definitions

[1]There is no limitation on the slope of the lines. This therefore covers Stigler's case of nickles and dimes being perfect substitution.

of marginal rates of substitution of one good for another, the shape of the curves on the indifference map indicates only substitution or the lack of it. Under the utility analysis it indicated both substitution and complementarity and much confusion resulted. Here the shape of the curve is limited only to the substitution relationship. This of course does not answer the question of what shape curve corresponds to a particular degree of substitutability, but it does show how substitution is the essence of the indifference system.

Ordinarily in this case the consumer will find the two goods indistinguishable in satisfying his wants. If the goods are physically indistinguishable, then they are for all intents and purposes the same good, but it is not absolutely necessary that the goods be physically indistinguishable to be perfect substitutes.[1] The case of nickels and dimes made previously is a case in point. Furthermore it should be pointed out that the relationship in the perfect case is reversible. If Y is a perfect substitute for X then X must be a perfect substitute for Y.

In the above discussion interest has been focused on those cases in which a fall on the price of X affects the demand for Y. But it is very possible that a fall in the

[1]This point is open to question. It may be argued that nickels and dimes are not perfect substitutes, and that where perfect substitutes occur they are really the same good and therefore physically indistinguishable.

price of X can have no influence at all on the demand for Y. This may occur if the income and substitution effects on the demand for Y are negligible or if not negligible, go in different directions and exactly counterbalance each other so that there is no difference between them. It is probable that many of the goods which economists have treated as independent come under the case where the income and substitution effects are negligible. But there also must be a few which come under the second case. It is possible in many cases that substitution in favor of other commodities comes about at the expense of close substitutes, but also there is probably a good deal of mild substitution which is present but prevented from showing itself because it is offset by income effects.

This is the theory of related goods as worked out by Hicks. It is a theory which applies to effects on individual demand, but by the process of additivity it becomes applicable to market demand as well. A fall in the market price of X affects the demand for Y through the income and substitution effects. As before the final direction of these effects is determined by the cumulative effects which may or may not cancel each other, but again here the possibilities of counterbalancing and cumulation are very numerous. For example X and Y may be complementary for some people and substitute for others. These individual effects then must be weighed against each other. In final analysis goods for the group

as a whole will still be regarded as complementary if the total substitution effect (which itself is the result of the counterbalancing of individual substitution and income effects) increases the demand for Y when the price of X falls. Goods may be regarded as substitutes if the reverse holds true.

It is seen then that in its connection to the theory of consumer demand, the important aspect of the new theory is the negative distinction which it makes in discarding indifference analysis. It is no longer directly dependent on this tool. But it is for this very reason that the indirect dependence should not be overlooked. It may be impossible to compare on the indifference map the marginal rates of substitution between two goods and money, but it is possible to establish the marginal rate of substitution between one good and money by this means, and since the new definitions are based on the latter concept, there is an indirect dependence. Furthermore in so far as the new theory compares income and substitution effects, even though the comparison is in words, there is a dependence on the indifference analysis. Therefore, as it was in the earlier development, the theory of related goods is still a special aspect of the theory of consumer demand, but the relationship between the two is much less close. This should make it much easier to distinguish them.

CHAPTER V

APPLICATIONS

We have spent much time in examining the indifference technique as it is used directly in the theory of consumer demand and indirectly in the theory of related goods. There remain now to be examined the various applications which can utilize the indifference technique. These fall into two broad classifications, those which limit themselves to the main theory of consumer demand only, and those which rely on the somewhat debatable concept of consumer's surplus.

Applications in the Theory of Consumer Demand

1. The Supply Curve for Labor. The number of hours a laborer will work per week is determined by the attractiveness to him of the money income received from working and the leisure which he must give up. This may be considered by means of indifference analysis where an ordinal preference scale is assumed in which the individual is indifferent as to quantities of income and hours of leisure. In Diagram 23 leisure is represented along the X axis and money income per week on the Y axis. Leisure is measured along OX up to point R which represents the total hours per week which may be

divided between leisure or working.[1] PR, P_1R, P_2R, etc., are wage lines.[2] Since the total number of hours is fixed, they measure all possible combinations of leisure, work, and income with a given maximum possible income. As in our previous argument the laborer will seek that combination of income and leisure which will put him on the highest possible indifference curve. This will occur where the wage line is just tangent to the indifference curve, point Q on indifference

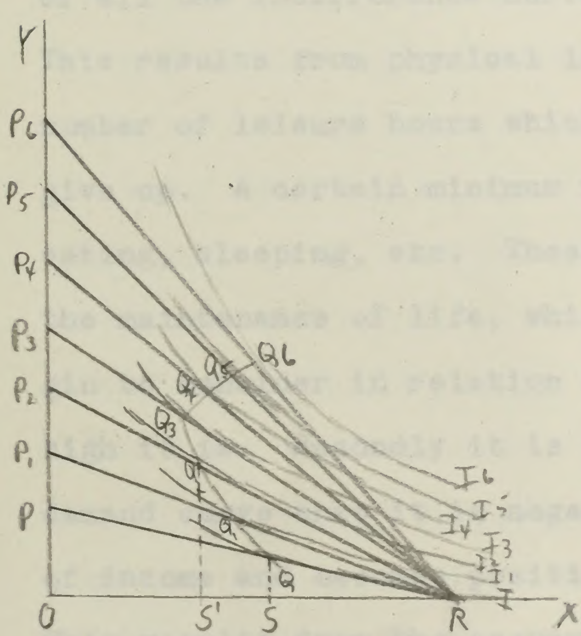

DIAGRAM 23

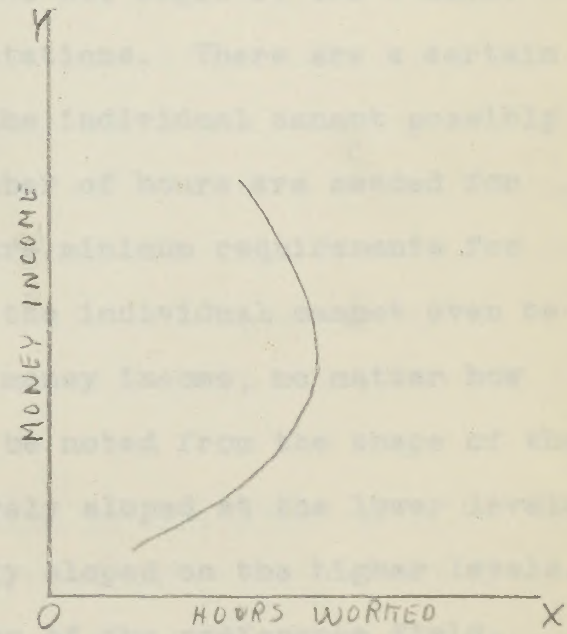

DIAGRAM 24

[1]In this case since total hours are limited, those hours not spent in leisure will be spent in work. Work may be considered as negative leisure.

[2]These are called wage lines because the slope of the line indicates the wage rate per hour which the individual receives by giving up leisure.

curve I. At this point OS hours will be devoted to leisure and SR hours to work. Similarly if the amount of income is allowed to vary, different divisions between leisure and work will be obtained at each point of equilibrium, Q_1, Q_2, etc. If we now connect the points of equilibrium we obtain a demand curve for leisure in terms of income (also for negative leisure, or work). This takes the form $Q\ Q_6$.

Several points are to be noted from the diagram. First of all the indifference curves do not begin at the Y axis. This results from physical limitations. There are a certain number of leisure hours which the individual cannot possibly give up. A certain minimum number of hours are needed for eating, sleeping, etc. These are minimum requirements for the maintenance of life, which the individual cannot even begin to consider in relation to money income, no matter how high it is. Secondly it is to be noted from the shape of the demand curve that it is negatively sloped at the lower levels of income and becomes positively sloped on the higher levels. This results from the assumption of the preference field, and it has been assumed in just this manner to indicate that the laborer will be attracted by the higher income and perhaps give up some of his leisure. But beyond Q_3 he will be more reluctant to give up his leisure in return for higher income, and will work less hours at the higher wage rate. This perhaps suggests that the higher the income the more

leisure will be needed to spend and enjoy that income. At any rate it is a possible suggestion of the explanation of the long decline in the hours of work which the current trend has indicated.[1]

It is just one step further to derive from the system of indifference curves the individual supply curve for labor. Since the hours of work are considered negative leisure, all that is necessary is to construct a scale in which hours of work appear along OX positively. This could not be done previously because hours of work are generally substituted for money in a direct ratio. In other words the indifference system could not be assumed with the curves having a diminishing marginal rate of substitution throughout. But if we now construct the new scale the result will be a reversal of the demand curve for leisure or the customary supply curve for labor shown in Diagram 24.

2. Effects of Different Taxes on Individual Effort. The question here is to what extent will individual effort be affected by the different types of taxation.[2] As in the previous case we assume a preference field as between hours of

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[1]This is by no means the only explanation. Increased productivity, a better standard of living, higher real wages, these are all to be considered, but in so far as they are indicated on the indifference system they will influence the curves toward the situation in Diagram 23.

[2]See Otto von Mering, The Shifting and Incidence of Taxation (Philadelphia: The Blakiston Company, 1942), pp. 111-114.

leisure and earned income. The line PR (Diagram 25) now indicates the market price at which the exchange of leisure for earnings takes place before the imposition of a fixed tax. Q, the point of tangency to the indifference curve, is

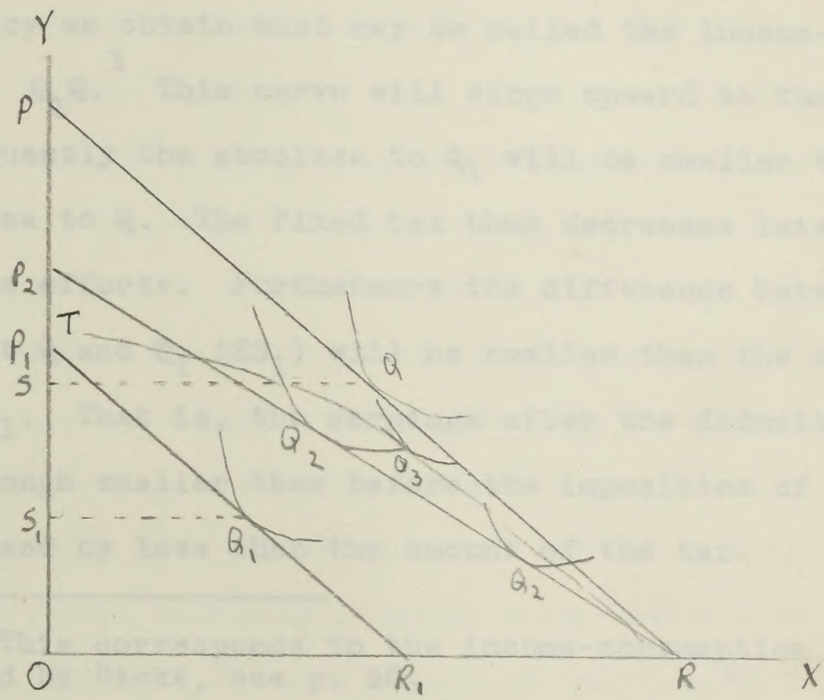

DIAGRAM 25

the equilibrium position before the tax is imposed.

Let us now levy a fixed tax equal to PP'. Automatically this reduces the amount of income the individual can earn. But at the same time it also reduces the maximum hours of leisure, for part of the total time available must be devoted, after the imposition of the tax, to the earning of the tax.[1]

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[1]*Ibid.*, footnote, p. 112.

If the entire tax is earned, the market price of the exchange of leisure for earnings will remain unchanged and the price line will be shifted to P_1R_1 by the exact amount of the tax. The new position of equilibrium will be Q_1 where the indifference curve is tangent to P_1R_1. Now joining the points of tangency we obtain what may be called the income-leisure curve, Q_1Q.[1] This curve will slope upward to the right.[2] Consequently the abscissa to Q_1 will be smaller than the abscissa to Q. The fixed tax thus decreases leisure and increases efforts. Furthermore the difference between the ordinate at Q and Q_1 (SS_1) will be smaller than the amount of the tax PP_1. That is, the earnings after the deduction of the tax though smaller than before the imposition of the tax are decreased by less than the amount of the tax.

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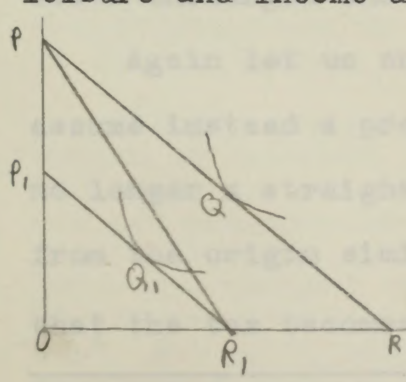
[1]This corresponds to the income-consumption curve developed by Hicks, see p. 58.

[2]A tax of a fixed amount may be considered as a negative gift. If the individual were originally at Q_1 and suddenly were given a gift equal to the amount of the tax, then with an unchanged market price of leisure, he would divide his leisure and income at point Q so as to increase the amount of leisure and at the same time have more money to spend. For this reason the curve slopes upward to the right. See von Mering, op. cit., p. 112. This is similar to the income effect developed by Hicks, but there is a definite limitation on the position of the indifference curves, that is, the slope of the income-consumption curve, imposed by the conditions of the problem.



Let us now assume that a proportional income tax is levied instead of the fixed tax. The new price line appears as RP_2, showing that as income increases the amount of the tax becomes greater by the fixed proportion and the amount of leisure less. The actual tax at any point is measured by the vertical distance between RP_2 and RP. The effect of this tax on effort depends on the position of the indifference curves and it is generally impossible to say what the position will be. The point of tangency Q_2 may fall to the right or to the left of Q either increasing or decreasing effort. What is involved here is the substitution effect between leisure and income as a result of an increase in the market price of exchange of the two. The direction of this effect will be indicated by the direction of the price-consumption curve QQ_2 and as we saw previously there is no limitation to the slope of this curve.[1] In this situation particularly it is impossible to say what the slope will be. We may conclude then that a proportional income tax may increase, decrease, or leave unchanged the effort of the individual.

Again let us change the conditions of the problem and assume instead a progressive income tax. The price line is no longer a straight line, but now becomes a curve convex from the origin similar to RT. This indicates, of course, that the tax becomes greater, the greater the income and

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[1]See page 62.

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[1]See page 82.

this is borne out by the increasing vertical distance between RT and RP as income becomes greater. Now if we make the assumption that there are points of equilibrium along this price line, like Q_3, it becomes apparent that as the tax becomes greater the abscissa to the point of equilibrium becomes less. In other words the more progressive the tax the greater the effort required.[1]

3. The Effect of Taxation on Individual Welfare. The problem here is whether or not the individual is better off with a commodity tax or a personal income tax. Obviously if the individual is to be taxed he will not be as well off as if he were not taxed, but how will individual welfare be affected by each of these taxes? Will he be on a higher indifference curve if the tax is levied in the form of an income tax or will a commodity tax place him on a higher curve?[2]

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[1]It may very well be that the tax at the higher or even the lower levels of income (depending on the rate of progressiveness) is large enough to stifle incentive so that the increased effort will not be put in. This of course would not appear on the above diagram where we consider effort negative leisure, but it is a limitation on that assumption which should not be overlooked even though it must be used in our analysis.

[2]This is based on the discussion by F. M. Joseph, "Excess Burden of Indirect Taxation," Review of Economic Studies, Vol. VI, 1939, pp. 226-227. Joseph's discussion attempts to analyze and measure the monetary loss to the treasury by the imposition of a commodity tax as opposed to an income tax. This utilizes the concept of consumer's surplus. I have therefore altered the discussion to avoid for now this concept, but my discussion is essentially the same as his. The consumer's surplus aspect of this will be treated below.

The problem is analyzed in Diagram 26. In this figure OX represents the commodity taxed and OY the individual's money income. The ordinal preference field is assumed as between Y (say, cigarettes) and money. The individual's money income will be equal to OQ. The price line will be QR. Any point on it represents the amount of income given up divided by the quantity. This of course equals price, but it is also equal to the tangent of QR or tan a. Before the imposition of the tax, QB cigarettes will be consumed and AB spent. The individual will be at equilibrium at A where the price line is tangent to the indifference curve.

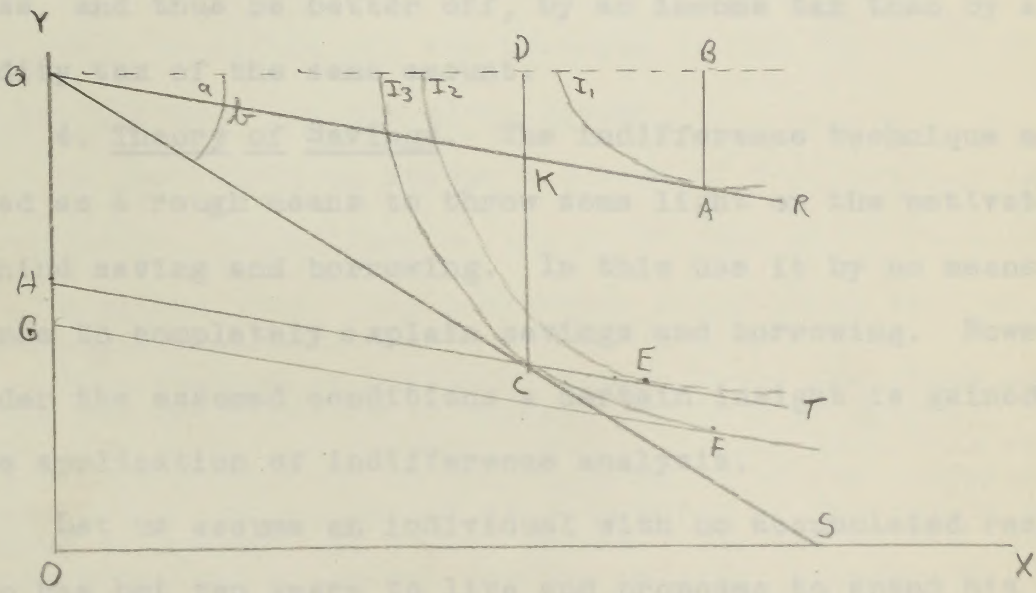

DIAGRAM 26

Now if a commodity tax is imposed on cigarettes, the price will rise to QS. The individual will be at equilibrium at point C where the indifference curve touches the new price

line. At this point he will buy QD cigarettes at a total cost of CD out of which KC is paid to the government as a commodity tax. Let us assume that the same amount of tax had been taken instead in the form of a personal income tax. The price of cigarettes remains unchanged, but income is reduced by the amount QH = KC. The new price line is HT through C but parallel to QR, and the point of equilibrium is at E where the new price line is tangent to curve I_2. Since E, the point of equilibrium, is on curve I_2 which is higher than curve I_3, on which C the previous point of equilibrium was located, it follows that the taxpayer will lose less, and thus be better off, by an income tax than by a commodity tax of the same amount.

4. Theory of Savings. The indifference technique may be used as a rough means to throw some light on the motivation behind saving and borrowing. In this use it by no means pretends to completely explain savings and borrowing. However, under the assumed conditions a certain insight is gained by the application of indifference analysis.

Let us assume an individual with no accumulated resources, who has but two years to live and proposes to spend his income of the two years in such a way that at the end of the period he leaves nothing for his descendants. Let it also be assumed that at any time he can borrow or lend unlimited amounts of money at rate of interest i. Then the question at hand is

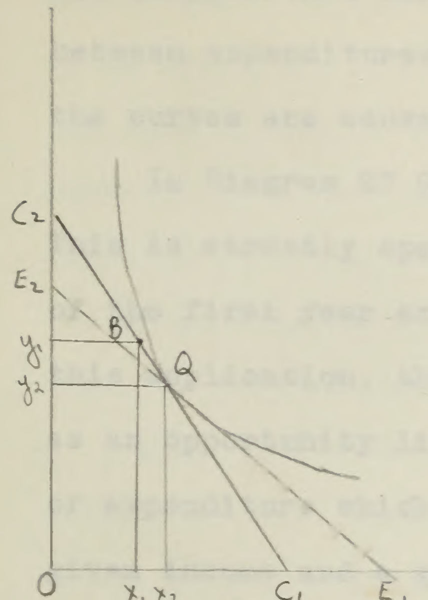

DIAGRAM 27

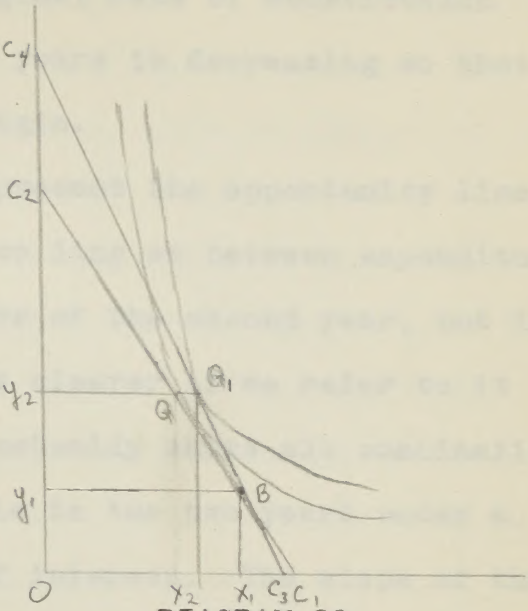

DIAGRAM 28

how shall he divide his expenditures in the two years so as to get the most for his given income.[1]

As previously we assume a system of indifference curves. X represents expenditure of the first year and Y expenditure of the second year. The preference field shows the indifference

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[1]The essence of this application under the given conditions was developed by I. Fisher, The Rate of Interest (New York: The Macmillan Company, 1907). Fisher used this as a "first approximation" in the explanation of the preference of present over future income as determined by the supply and demand of present and future income. His use of the indifference technique (except for the utility concept) is similar to that given above and his results are substantially the same. See Appendix, Chapter 7, pp. 387-392. See also the later expansion of this application by Fisher, The Theory of Interest (New York: The Macmillan Company, 1930), Ch. 10, pp. 231-258, particularly p. 255.

Boulding also has worked up the same application. His is taken from Fisher, but he has added a simple algebraic explanation which may prove helpful. See R. E. Boulding, Economic Analysis (New York: Harper Brothers, 1941), pp. 669-673.

as to combinations of expenditures in the two years. Like all indifference curves the marginal rate of substitution between expenditures of the two years is decreasing so that the curves are convex to the origin.

In Diagram 27 C_1C_2 will represent the opportunity line. This is strictly speaking a price line as between expenditure of the first year and expenditure of the second year, but in this application, the meaning is clearer if we refer to it as an opportunity line, for it actually shows all combinations of expenditure which are possible in the two years under a given income and a given rate of interest. The slope of the line depends on the rate of interest. If, for example, income is \$1000 the first year and \$2000 the second year and the interest rate zero, the individual can borrow \$2000 the first year and spend \$3000 or he can loan out \$1000 the first year and spend \$3000 the second year. With a zero interest rate $OE_1 = OE_2$, and E_1E_2 becomes the opportunity line when i is zero. On the other hand if i is 10 per cent the individual can lend out \$1000 the first year and receive interest on this to spend in the second year making a total expenditure of \$3100, OC_2. Or he can borrow \$1818.18 the first year which together with his income will allow him to spend \$2818.18, OC_1, the first year. The second year he will spend nothing but out of the \$2000 income he will pay back his loan of \$1818.18 plus the interest at 10 per cent of \$181.82. The

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opportunity line in this case will be C_1C_2 indicating possible expenditure combinations in the two years at an interest rate of 10 per cent. If the individual neither borrows nor lends, but spends each year his exact income, then regardless of the interest rate, his given yearly income is an expenditure possibility. We may say then that the opportunity line changes its slope depending on the rate of interest, but whatever the rate, it rotates on the point of yearly incomes, point B.

From the technique of the indifference analysis, the individual will seek that combination of expenditures which is on the highest indifference curve touched by the opportunity line; that is, he will seek the combination which will give him the greatest satisfaction consistent with opportunity possibilities. Point Q, therefore, where the opportunity line C_1C_2 is just tangent to the indifference curve will indicate the best combination on that line. With the given yearly incomes and a given interest rate of 10 per cent the individual will spend Ox_2 the first year and Oy_2 the second year. He will have to borrow the sum x_1x_2 the first year and pay back the sum y_1y_2 the second year. y_1y_2 however includes the interest on x_1x_2 so that the total amount of interest paid will equal $y_1y_2 - x_1x_2$.

It should be particularly noted that the interest rate is fixed. This however should in no way influence the instantaneous establishment of the indifference curves for they

merely represent the level of individual indifference to given combinations of expenditure. The slope of the indifference curve at any particular point represents the individual's marginal rate of substitution between expenditures of two years. Actually according to the conditions of the problem the process of seeking an equilibrium point amounts to just equating two independent factors, one the given market rate of interest, the other the assumed marginal rates of substitution.[1] This is important for it is very easy to fall into the erroneous belief that the indifference curves are influenced by the given interest rate. This, however, is ruled out by the hypotheses.

It is evident from Diagram 27 that an increase in the interest rate will make the individual worse off. The opportunity line C_1C_2 must rotate on B, the given yearly income combination. If the interest rate is increased, C_1C_2 becomes steeper. Therefore the new equilibrium point must lie on some curve below the one given in the diagram. The reason for this is that the individual's income is concentrated mostly in the second year and as a result he borrows the first year and repays in the second. The increased rate of interest will make him borrow less, but it will also lower

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[1]Boulding refers to this as the "rate of time substitution of expenditure" or "time preference." See <u>Economic Analysis</u>, <u>op</u>. <u>cit</u>., pp. 624-675.

the terms on which he can borrow so that the net effect will be a decline in individual well-being. On the other hand if the greater part of the total income is concentrated in the first year (Diagram 28), the individual has an excess of income that year and becomes a lender. An increase in the rate of interest therefore makes him better off.

It is apparent from this reasoning that there are three factors governing the position of the individual, the indifference curves, the interest rate, and the distribution of income. If the interest rate and indifference curves are given, then the distribution of income will decide whether the individual is a lender or borrower the first year. In other words, the individual's impatience to possess money now or in the future would be directly dependent on the distribution of income. On the other hand once the distribution of income is determined, and from this it is decided whether the individual will be a borrower or lender, the amounts borrowed or loaned, and the amounts spent each year will depend upon the rate of interest. In this case the rate of interest determines the individual's preference for money in the present or in the future, and it also determines individual well-being.

This application, within the scope of its limitations, shows some of the relationships between the rate of interest, preference for money, and individual well-being. By its very nature it is limited to two periods of time. They may cover

two years, or may be considered as early years or later years. If three periods of time are considered, then it becomes necessary to construct a three dimensional diagram. Beyond three periods diagrams fail, but enough can be learned from the simple application to make the extended analysis by means of algebra.

Applications Involving the Concept of Consumer's Surplus

As we saw in our previous discussion, the restatement of the concept of consumer's surplus in terms of indifference analysis attempted to translate gains or losses into a numerical monetary measure. This procedure was justifiably criticized. However, aside from the criticism, certain applications have been made by those who saw merit in the restated concept, and to complete our previous discussion, several of these will be given.

5. <u>Tax Yield to the Government</u>. In the third application (Diagram 26) it was seen the individual fared better by the imposition of an income tax as compared with a commodity tax of the same amount. Let us now look at this from the opposite point of view and see how the government is affected by the imposition of these taxes. It is at once obvious that the government loses by the imposition of an income tax as compared to a commodity tax of the same amount, but governments

are interested in monetary measures of losses and gains, so that it becomes necessary to measure in terms of money the additional loss to the government by the imposition of different taxes of equivalent yield.[1]

In Diagram 26 the imposition of a commodity tax raises the price, moving the price line from QR to QS and the point of equilibrium from A to C. At this point the total amount paid is CD of which the yield to the government is CK. When the equivalent yield is taken by means of an income tax, price remains the same, but total income is reduced to OH. Individual equilibrium moves to point E which is on a higher indifference curve than point C. This means that the government could have obtained an additional yield by substituting an income tax for the commodity tax without imposing any extra burden on the consumer.

Under the commodity tax the individual was at equilibrium at C on indifference curve I_3. If we draw a new line parallel to the price line HT under the income tax, but tangent to the indifference curve at the same level of C, I_3, we can then obtain a measure of the additional yield which the government could have got. We can say now that a commodity tax yielding CK is the same to the individual as an income tax yielding QG. The additional tax which the government could have got

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[1]Joseph, "Excess Burden of Indirect Taxation," op. cit., p. 226.

by imposing an income tax thus is measured by GH.[1]

The size of GH depends on the relationship of the two goods. As was seen previously the only relationship to be derived from the indifference curves is the presence of or lack of substitution. In the case of complete lack of substitution the marginal rate of substitution at a particular point would be zero. If for example this occurred at C, HC would be tangent to the same indifference curve as QC[2] (Diagram 29). Therefore there would be no difference in the burden to the individual consumer between an income tax or a commodity tax and hence no excess loss to the government.

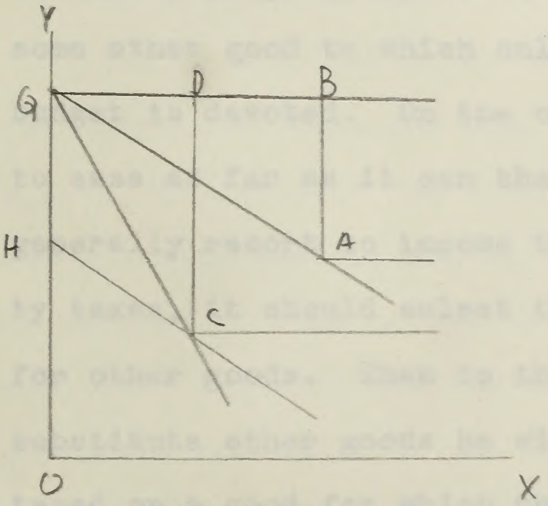

DIAGRAM 29

It would seem to follow from this that where the government wants to drain purchasing power and a commodity tax is used, it should be levied on

[1]In terms of the consumer's surplus discussion, GH is equivalent to the "price compensating variation," BB_1 in Diagram 14, Chapter III.

[2]C would be the point of complete lack of substitution. Pareto concluded that the L-shaped curves represented extreme complementarity. Hicks has shown that the only relationship to be derived from the indifference curves is that of substitution, therefore I am assuming that at c the L-shape curves indicate complete lack of substitution. This has not been sufficiently validated, but I utilize this assumption to bring out the point of application.

those commodities which do not readily substitute for money (and hence other goods); otherwise there is loss to the government to the extent that additional taxes could be collected without imposing further burden on the consumer. This also suggests that those commodities should be selected which customarily make up a large part of the individual budget. The very size of the budget devoted to the consumption of a particular good indicates that the substitution between that good and money is probably (not necessarily) lower than that of some other good to which only an insignificant part of the budget is devoted. On the other hand if the government desires to ease as far as it can the burden of the consumer, it should generally resort to income taxes, but when it does use commodity taxes, it should select those commodities which substitute for other goods. Then to the extent that the consumer can substitute other goods he will be better off than if he were taxed on a good for which there are no close substitutes.

6. <u>The Problem of Index Numbers</u>. The indifference analysis may be applied to the problem of index numbers. This application follows directly from the later developments of the consumer's surplus concept, but it is not directly dependent on this concept. It is rather, as Hicks explains, that the discussion of consumer's surplus has served as the connecting link to the theory of index numbers.[1] This, I think, will

[1]See above, page 90.

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[1]See above, page 90.

become apparent as the discussion proceeds.

The problem of index numbers is one phase of General Welfare Economics. Simply stated the problem is this: Given two consumption periods, is it possible to establish that the individual's standard of living has risen or fallen between the two periods, and is there a measure of the change in the standard of living? Theoretically we are chiefly interested in the first part of the problem, but we cannot overlook the measurement factor.

Let us start with two periods, A and B. In period A the individual consumed $q_o q_o'$ of two commodities at prices $p_o p_o'$. In period B the individual consumes quantities q_1 and q_1' at prices p_1 and p_1'. Now we have two commodities of which different quantities are consumed in different periods. Indifference curves can be arranged between the different commodities, but in order to make any progress we must make the same assumption which we made in the discussion of consumer's surplus, namely, that tastes remain unchanged in each period.[1] In order to make comparisons it is necessary to use one system of indifference curves; otherwise changes in the standard of living are meaningless.

Given constant tastes, then the quantities and prices allow us to establish the consumer's income and equilibrium position. In Diagram 30, q_o is consumed at price p_o and q_o'

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[1]See above, page 86.

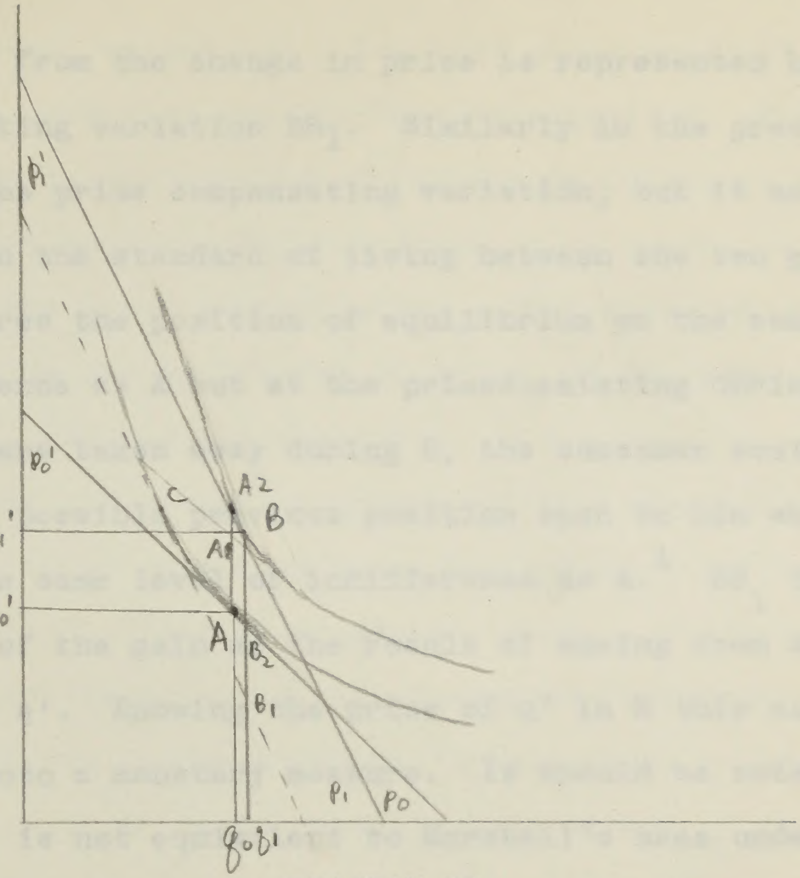

DIAGRAM 30

at price p_o'. Consumer's income is then determined by line $p_o p_o'$ and his equilibrium position is established at A. Similarly in the second period $p_2 p_2'$ represents consumer's income with the equilibrium position at B. It is at once obvious that position B is on a higher indifference curve than A and that therefore the individual has a higher standard of living in B.

But is there a measure of this change in standard? Referring back to Chapter III, Diagram 14, we see that the changes in the two periods are tantamount, under the assumption of constant taste, to changes in price. The measure of

the gain from the change in price is represented by the price compensating variation BB_1. Similarly in the present diagram BB_1 is the price compensating variation, but it measures the change in the standard of living between the two periods. It compares the position of equilibrium on the same level of indifference as A but at the prices existing during B. Thus if BB_1 were taken away during B, the consumer would move to the best possible previous position open to him which is at C, on the same level of indifference as A.[1] BB_1 is thus the measure of the gain as the result of moving from A to B in terms of q'. Knowing the price of q' in B this can be converted into a monetary measure. It should be noted however that BB_1 is not equivalent to Marshall's area under the demand curve. This confusion was straightened out by Henderson as we saw in our discussion of consumer's surplus. But it is a measure of the gain as a result of the change in price, and it therefore permits us to measure the change in the standard of living.

Other measures of the change in standard of living have been developed which also evolve on measurements from the consumer's surplus diagram. Most of these are mathematical measures; therefore I will present them without their

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[1]J. R. Hicks, "Consumer's Surplus and Index Numbers," <u>Review of Economic Studies</u>, Summer, 1942, p. 127. Compare <u>Hicks'</u> diagram with Diagram 14 in Chapter III.

mathematical background. Two well-known measures are the Laspeyre variation and the Paasche variation. The Laspeyre variation is the measure of the change in income which would enable the same quantity of goods to be purchased (the quantity in the first period) after prices have changed. This is measured by AA_1 in Diagram 30 or AA_2 in the consumer's surplus diagram. The Paasche variation, on the other hand, is the measure of the change in income which allows the second set of goods to be purchased under the prices existing in the first period. This is indicated by BB_2. Still a third measure is what Hicks calls the equivalent variation. It measures the change in income which if it took place on the basis of the prices in A, would have the same effect on the level of satisfaction as is produced by the change in prices from A to B. This is measured by AA_2. All these measures can be expressed mathematically and also can be quantitatively compared.[1] For our purpose however, it is sufficient to show that they exist and can be represented geometrically on the indifference map.

The above are a few of the important applications of the indifference analysis. They do not by any stretch of the imagination exhaust all the possibilities, but they give

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[1]For mathematical discussion see ibid., 129-137. See also R. Frisch, "Problem of Index Numbers," Econometrica, 1936, pp. 1-38.

us an adequate idea as to how some practical value may be derived from the indifference analysis. From a theoretical point of view those applications which stand on the customary indifference technique are to my mind the more substantial. Those which depend on the consumer's surplus concept, while valuable perhaps from a descriptive point of view, contain a theoretical weakness which cannot be overcome. To this extent then they should be accepted with limitation. Moreover the same result can be obtained without the numerical measure of gain. It would seem wise therefore to concentrate on the direct use and eliminate the doubtful one. As explained previously this is, in the main, the method I have followed.

CHAPTER VI

EVALUATION

In the preceding pages we have spent much time examining the development and use of indifference curves in demand analysis. There remains now the question of evaluation, evaluation of the technique as a tool, and evaluation of the theory in which the tool has been used. As was seen in the development, it is inevitable that both go hand in hand.

The theory of consumer demand is but one phase of the more inclusive theory of value. It has been important to us because it was in this phase that the indifference analysis was introduced as a technique which held promise to make a progressive contribution theoretically and descriptively. Whether or not it has added anything useful in these respects is the main problem of evaluation. The criteria for judgment of this problem lie in the consistency of the theory in which the technique has been used and in the ability of the theory to illustrate the truth, that is, to conform to reality. So far as the logic of the theory is concerned no difficulty presents itself. If we take the theory of Hicks as the latest and most generally accepted constructive theory of consumer demand (which utilizes the indifference device), then there is no problem about the

logic of his theory. For that matter there was no question about the logic of Pareto or Fisher on whom Hicks built. The work of each of these in itself was entirely consistent. It was merely that they were lacking in their ability to conform to reality. Additional limitations, new concepts, etc., were needed to illustrate the truth. These were largely supplied by Hicks so that in final analysis the problem reduces to an evaluation of the ability of Hicks' theory to explain what actually takes place in society.

Economists are very familiar with the difficulties inherent in the social laboratory as opposed to the physical laboratory. It is because of these difficulties that they have been obliged to devote so much of their time to theory. The problem of verification of theory still remains one of the greatest stumbling blocks in the science; indeed it is because of this that theory rather than law occupies so great a portion of the subject matter. But this is not an absolute obstacle. It is still possible to build better and better theories, to reason from assumptions which more nearly correspond to reality and which hold greater promise for verification. This was particularly apparent in the development of the indifference analysis out of the utility theory. Utility as an assumption presented a fairly reasonable foundation, but as we saw, it was later proved to be incapable of verification. The history of indifference analysis, if it has done nothing more, has pointed

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this out, and as a result there is today general unanimity that the assumption of utility as a measurable magnitude does not conform to actual fact.

It is fitting now to raise the same doubts about Hicksian theory as were raised about Marshallian utility theory. Can the assumptions of this theory be verified to the extent that the results of the logic built on them reasonably portray the truth? The assumptions with which we must be concerned are those which Hicks developed to take the place of the doubtful concepts in utility theory, in particular those connected with the principle of the marginal rate of substitution.

It will be remembered from the early discussion that the principle of marginal rate of substitution was developed to avoid the arbitrariness of marginal utility. The marginal rate of substitution of X for Y was defined as the quantity of Y which would just compensate the consumer for the loss of the marginal unit of X. In Marshallian terminology, if Y is money, then the marginal rate of substitution of X for Y is equal to the price.[1] But Marshall assumed the marginal utility of money to be constant. Hicks makes no such assumption; so that actually the marginal rate of substitution of X in terms of money is independent of price.[2] Income effects can enter and the

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[1]Hicks, <u>Value and Capital</u>, <u>op</u>. <u>cit</u>., p. 20.

[2]Above page 49; also footnote on page 147.

advantage of the new theory was precisely the fact that it treated these effects.

In view of the criteria of evaluation which have been set up a logical question presents itself. If the marginal rate of substitution is not equal to price, then what is it and how can it be verified? It appears to me that there is no satisfactory answer. The marginal rate of substitution is a concept similar to the utility concept. Logically it does not contain the objectionable measurement feature of the utility concept, but aside from this if an individual's marginal rate of substitution between X and Y is to vary, apart from the price of X in terms of Y, then the concept still retains an introspective aspect. It is true that this aspect is not a measurable one, but nevertheless it remains mental and as such defies any possible verification. This I think is particularly well brought out in Hicks' definitions of related goods. They are framed in terms of the effect of a change in the marginal rate of substitution of X for money on the marginal rate of substitution of Y for money. If the price of Y is given as constant, then the condition of relatedness will affect the marginal rate of substitution of Y for money so that the marginal rate of substitution of Y money "is kept equal to the price of Y."[1] This certainly infers that the

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[1]Hicks, op. cit., p. 46.

marginal rate of substitution is a concept apart from price and since it is something which evidently exists within the individual, it must be purely introspective.

Hicks considers the principle of diminishing marginal rate of substitution as "a positive change in the foundation of the theory."[1] In a certain sense it is, for it permits the establishment of continuous indifference curves convex to the origin on the basis of a new concept. But the change is not as radical as his language would infer.[2] In comparing his theory with that of Pareto or Fisher it is very evident that there are close similarities, even though the latter utilizes the principle of marginal utility. In essence, even with the new principle, Hicks' results are merely a transformation of the utility concept into other language. It is true that in the process of transformation certain objectionable features of the old language have been removed, and other new and beneficial features have been added, but this hardly makes the change positive. For the new results, while admittedly better than the previous ones, still defy objective verification. The theory is still in the realm of mental speculation. As such it cannot be empirically coincided with human behavior and therefore defies objective refutation. Now it may very

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[1] Ibid., p. 21.

[2] See on this point O. Morgenstern, "Professor Hicks on Value and Capital," Journal of Political Economy, June, 1941, p. 366.

well be that all demand theory, by its very nature, must be limited to this realm. Indeed under the present state of knowledge, this appears to be so, and if it is, the problem of demand will always remain as theory and never pass into the sphere of law. In this sense assumptions such as Hicks makes are necessary and, within limits, useful. But it is not inconceivable that some new social or psychological technique will be developed which will do away with these assumptions. An advance of this sort would then be considered a change in basic foundation. Until this appears, however, economists must make use of devices similar to those used by Hicks. It is only through such devices that the total understanding of economic phenomena will be increased. The greater the understanding, the greater will be the probability that eventually theory will become law.

In the light of this, then, Hicksian theory can serve a definite purpose. But in the same light, the utility analysis is also technically meaningful. This, therefore, precludes the complete abandonment of the utility analysis. It attacks essentially the same problems as does indifference theory. Its language is not as logically precise nor does it go as deep into particular phases, but it does possess an unparalleled simplicity which permits description of unexcelled clarity. As long as the points of superiority and inferiority of both theories are recognized they can be used parallel to

each other. Indeed this is borne out by the fact that while an increasing number of authors are using indifference analysis, they are not dropping the utility approach. Some are using one to check the results of the other. Others are using the indifference analysis to clarify particular points for which it is better suited. At any rate both have become tools of the economist and are accepted as such.

Since this is the situation, it is advisable to clearly separate the two theories in the important aspects. As has been pointed out previously, the chief differences lie in the original assumptions. The assumption of utility infers a measurement. The assumption of a preference field merely indicates levels of equal preference which are superior to one another. The exact extent of superiority is entirely unnecessary, for the theory can be built without any reference to this measure. But this is all basic. Beyond this there is the question of income effects. The underlying assumption of the utility theory is the constant marginal utility of money. It was this which prevented Marshall from coming to the income effect and it is this which sharply differentiates the two, particularly in a descriptive sense. However one may wish to stress the importance of the income effect, common sense tells us that it does exist, and to the extent that the indifference analysis treats this effect, it is superior to the utility theory. In his text Hicks makes quite a bit of

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the income effect, yet he is very careful to portray it in its proper relative position. This he does by translating Marshall into his own language. He shows how Marshall's constant utility of money is equivalent in his own terminology to the case where the income effect is swamped by the substitution effect. This usually occurs where changes are small and income effects are unreliable, and as these conditions prevail in the vast majority of cases Marshall's assumption is generally valid. It is for this reason that Marshall was able to establish an unequivocal theory of demand. But there are cases where this assumption does not fully conform to fact. It is here that indifference analysis fits the bill, for it treats those cases for which the utility theory is not adaptable and thereby presents a more accurate picture.

The doctrine of constant marginal utility of money has given rise to a great amount of discussion, much of it very unnecessary.[1] This is very true with respect to the concept of consumer's surplus. As we saw in our discussion a great deal of confusion was created by a lack of understanding as

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[1]Samuelson is particular to refer to the doctrine as the constancy of marginal utility of income. He points out that Hicks misunderstood Marshall's use of the word money, and that therefore even in Marshall's work the marginal utility of money is not intended to be constant with respect to everything. But Marshall definitely considered changes in the marginal utility of income as of the "second order of smallness" and therefore did not even approach income effects. See P. A. Samuelson, Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947), pp. 189-194.

to just what Marshall meant. This misunderstanding was fostered even further by Hicks' restatement of the concept. For Hicks the establishment of indifference curves allowed the restatement on what he considered more realistic grounds. In his pure consumer theory the constant marginal utility of money led to the investigation of income effects, which were completely lacking in Marshall's theory. Hicks felt that this also carried over to the concept of consumer's surplus, extending it beyond the case where indifference curves are merely parallel to each other. To the extent that income effects were lacking this was probably true, but it did nothing to revitalize the concept itself, for as was pointed out, there were defects in the concept, aside from the lack of income effects. Even after Hicks' restatement it is generally realized that nothing is to be gained from the concept. It would seem wise therefore to exclude it completely and avoid unnecessary discussion.

As for the theory of related goods here too much confusion still remains, particularly with the problem of complementarity. There is no question that the theory of Hicks is a definite improvement over that of the utility school, but there still remain many vital questions for which there is no clear-cut explanation. This leaves us with a theory which is not particularly useful. The reason for this may lie in the definitions of complementarity. Samuelson has pointed out that Hicks

gives two or more distinct and inconsistent definitions.[1] The mathematical one (which we did not go into) can be applied to the case where there are only two goods, but the verbal does not apply to this case. Indeed the results of the verbal definition point out the necessity for the third good in order that the concept make any sense. This is clearly an inconsistency which has not and should be cleared up. But even on the basis of the verbal definition some of the old problems still remain. It is still possible for any two goods to be both complementary and substitutable at the same time. The relationship between the two concepts and the limits of each are still confused. The question of degree is still ambiguous. Generally speaking the theory is still in rough form and leaves much to be desired.

But apart from this, the main theory of consumer demand which is based on the use of the indifference technique, is theoretically useful. Some of the applications have been examined above. No doubt many more will be developed. The work of Hicks is by far not the final answer, but even in its present form it presents a tool which lends itself to the problem of theoretical analysis. As such it deserves a place as a part of the economist's equipment.

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ABSTRACT

Indifference analysis is essentially an outgrowth of Marshallian utility analysis. In its early development it sought objectivity, but at the same time it still clung to the utility concept, for this was so firmly embedded in early thought that it was difficult to do away with it. However, the utility concept contained many inherent defects. Though based on a purely psychological magnitude and though theoretically precise, it defied practical measurement. A descriptive theory was built upon it, but it was impossible to verify that theory inasmuch as utility, as such, was purely subjective and afforded no tangible standard of measurement which could serve as a guide. With the development of the indifference system, part of this objection was overcome.

This was apparent, not so much in the work of Edgeworth, but particularly so in that of Fisher and Pareto. They both began with the utility concept. By utilizing the three dimensional device in a manner similar to that of Auspitz and Lieben, early predecessors, they established a utility surface. But they then did away with the third dimension, and there remained a series of curves in two dimensions which indicated constant levels of utility. These curves were assumed in the form of

an indifference system or map, and served as the starting point for a new theory.

The new tool presented a distinct advantage, for it apparently permitted demand theory to be objectivized. It was no longer necessary to have a cardinal measure of utility showing by how much one good was preferred to another. All that was needed was an assumed preference field indicating that one combination of two goods was preferred to another, more, or less, but not by how much. This disposed of the need for a standard of measurement since the strictly quantitative implications of the earlier theories were no longer present. Thus both Fisher and Pareto claimed complete objectivity. To a great extent they were justified in their claims, but not fully so, for as was pointed out by Hicks, utility still entered through the back door.

Current indifference theory as developed by Hicks proposes to do away completely to any reference to utility. Hicks examined previous theory, and systematically disposed of those concepts tainted in any manner by utility. In place of marginal utility he established the new principle of the marginal rate of substitution. In place of diminishing marginal utility he substituted the principle of diminishing marginal rate of substitution. The latter principle he considered a basic change in the very foundation of demand theory. His results are for the most part theoretically objective. The measurement

objection of the utility concept has been overcome, and in the process it has become possible to describe income effects. These were known to have existed previously but had never been treated. The income effect is the effect on consumer expenditure of a fall of price, which reacts like an increase in income. This is distinguished from the substitution effect, of a fall in price, which allows the additional increment to be spent in purchasing increased quantities of the good whose price has fallen. To the extent that Hicksian theory explains income effects without any reference to utility, it is superior. It not only goes further than utility theory, but it does so without any dependence on a measurable magnitude.

Hicks considered his new principles a positive change in the foundation of the theory. In a certain theoretical sense they are, for they dispose of previous weak points. But one cannot help notice the parallels between the work of Pareto and Hicks. It is very evident that the new theory is merely a translation of the former work into the language of the new principles. Additional features have been added. The results are definitely better. But basically the theories remain the same and what is most important is that they are both still within the subjective sphere.

Hicks hoped to have overcome this, but by setting up the concept of the marginal rate of substitution, as an independent entity, varying apart from price, he establishes a ratio which

exists solely in the individual mind. To be sure the problem of measurement is not involved, for how this ratio varies quantitatively is of little interest. But the very fact that it is a mental phenomenon deprives it of objective verification. This is particularly apparent from his definitions of related goods which are indirectly dependent on indifference analysis. Here the conditions of relatedness depend on the variations of marginal rates of substitution between two goods and money. These marginal rates of substitution are not equal to price. They are established by assumption only, and, while theoretically helpful, defy any verification. This means, of course, that the results can never be established empirically and that theory can never become law, if any such law exists. It may well be that all demand theory must remain within this subjective sphere. If this is so then it is wise to recognize its limitations.

In view of this Marshallian utility analysis attains a new value. If the measurement defect and the method by which the indifference analysis overcomes this, are both taken into consideration, it is possible to use the utility analysis with advantage. It certainly is less complicated, and attains a degree of clarity in explanation which is not inherent in the indifference analysis. Yet there are many problems for which it is not suitable, specifically those which deal with income effects. It would seem wise therefore, rather than to accept

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one as superior to another, to adopt both as tools and wherever possible to use one to check the other. This opinion seems to be borne out in actual practice, for this procedure is being followed more and more in current economic work. The indifference technique is becoming more familiar and more widely accepted, and at the same time utility analysis is losing none of its acceptance. Both are being used as parallels to each other.

The problem of related goods has been intricately bound up with the development of indifference theory. The very idea of levels of indifference between different combinations of goods suggested treatment of substitution and complementarity. At first this was done on the utility basis. Since the indifference map treated the level of utility of two goods together it was natural to ask how the utility of one was affected by changes in the utility of the other. Thus we find treatments by Fisher and Pareto, differing in many respects, but essentially the same. The peculiar part of both these treatments is that while both claimed objectivity in the central development of demand theory and in a limited sense attained it, whatever objectivity was attained was completely destroyed in the theory of related goods.

The main difficulty has been in the establishment of the exact nature of substitution and complementarity, and their relationship to each other. There is something in the concepts

which has eluded all who have treated it. The problem of complementarity in particular has presented great difficulty. And to a lesser degree this situation still prevails in current theory. The work of Hicks is an improvement over that of Pareto. Hicks performed a valuable service in showing how money entered, and how indifference theory is only indirectly helpful in the problem of related goods. But Hicks' theory is far from the final answer. His concept of money is vague. Many of the problems inherent in Pareto still remain in the work of Hicks, particularly the problem of degree. The exact nature of the concept has not adequately been explained and generally speaking the theory is still very rough and of little value. But this is entirely apart from the theory of consumer demand. The latter does have value in itself and should be considered separately.

Indifference theory can and does stand alone as an economic tool.

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About the Writer

Leon B. Levitan was born on September 13, 1917, in Boston, Massachusetts. His father, Nathan Levitan, was born in Poland, but came to this country at a very early age and was educated in the Boston schools. His mother, Mae Levitan, was American born and also had the benefit of an education in the Boston school system. The writer was the oldest of four children, three boys and one girl. He was also educated in the Boston school system, graduating from Boston Latin School in 1934. From 1934 to 1938 he attended the University of Maine at Orono, Maine, from which he graduated with the degree of Bachelor of Arts. His major interest during his undergraduate years was in the field of Economics, an interest which carried over into his graduate work. He was a member of Tau Epsilon Phi, an undergraduate fraternity, and is still active in the alumni organizations of that group. In 1943 he began graduate work at Boston University. The first phase of this work was completed in August 1945 when he was awarded the degree of M.A. in Economics. The following September this work was continued and will culminate in August 1948 with the awarding of the degree of Ph.D. in Economics.